IJGE

ISSN: 2146-9296
www.ijge.net

International Journal of Global Education-2013 volume 2, issue 4

7TH AND 8TH GRADE STUDENTS` GENERALIZATION STRATEGIES OF PATTERNS

Research Assistant Pınar GÜNER
Ondokuz Mayıs University, Faculty of Education, Samsun, 55160, Turkey
pinar.guner@omu.edu.tr

Assist. Prof. Dr. Esen ERSOY
Ondokuz Mayıs University, Faculty of Education, Samsun, 55160, Turkey
esene@omu.edu.tr

Research Assistant Tuğba TEMİZ
Van Yüzüncü Yıl University, Faculty of Education, Van, 65000, Turkey
tugba.temiz86@gmail.com

ABSTRACT
Pattern is a combination of shapes, sounds, actions or symbols in a specific order (Souviney, 1994). The definition of mathematics which is stated as pattern and system science (Goldenberg, Cuoco and Mark, 1998) and the universal language which is used to understand the relationships among these (Olkun and Toluk-Uçar, 2006) shows the importance of pattern in mathematics. Pattern is one of the main concepts that contribute to comprehend mathematical concepts, recognize mathematical relationships and interpret them correctly (Burns, 2000). Therefore, it is important to know the strategies that students used to reach generalization in patterns and how they think in this process in terms of teaching mathematics. The purpose of this study is to describe 7th and 8th grades students` ways of thinking related to pattern and investigate the generalization strategies. This research was carried out total 8 students attending the teaching program at 7th and 8th grades in a primary school in İstanbul in the term of 2012-2013. Of these participants, 4 were at 7th grade and 4 were 8th grade. Open ended 4 problems related to patterns were used as data collection tool. Besides, semi-structured interviews were made with the aim of exposing how students think while solving these problems. The collected data was classified through the generalization strategies in the related literature. As a result of the study it is seen that most of the students use guess and check or explicit strategies whereas few of them apply contextual and only one use addictive strategy. In addition, addictive strategies are usually used in near generalization whereas explicit strategies are used in far generalization.
Keywords: Pattern, generalization strategies, 7th and 8th grade students

Introduction

Mathematics is seen as the science of patterns and order (Steen, 1988) and looking for a pattern or regularity is one of the actions which are performed in mathematics on the whole (Orton, 1999). There are several definitions of patterns which have important place in mathematics. According to Guerrero and Rivera (2002), pattern is a rule among the elements of mathematical objects such as numbers, shapes. Pabic and Mulligan (2005) has defined pattern as spatial or numerical regularity whereas it has been defined by Olkun and Toluk-Uçar (2006) as a system which consists of the objects or shapes are recurring or ordered regularly. In addition, according to Orton and Orton (1999) pattern is an approach which conveys to algebra. According to Reys, Suydam, Lindquist and Smith (1998) patterns help students to develop the skills of calculating, putting in order and structure their thinking strategies. In addition they have important role in improving the skills of reasoning, communication, association and problem solving (Tanışlı and Özdaş, 2009). Patterns generally include the actions of counting, comparing, classifying, measuring, estimating and making symbolization and these processes make students` mathematical skills and knowledge meaningful (Fox, 2005).

Patterns are at the center of mathematical thinking and mathematical inquiry (Waters, 2004). The activities related to patterns are essential in terms of realizing mathematical relationships, understanding the system and logic of mathematics (Burn, 2000). Besides they constitute prerequisite for algebra and have a significant role in terms of development of it (Herbert and Brown, 1997). In general, these activities include searching for pattern, extending patterns, making pattern generalization (Lan Ma, 2007) and contribute to the skills of organizing data systematically, conjecturing and generalizing (Barbosa, Vale and Palhaders,2012). Generalization which is described by Dörfler (1991) as a tool and communication of thinking is one of essential aims of mathematics instruction (NCTM, 2000). At this point, patterns is a fundamental step in the formation of generalization (Hangreaves,Shorrocks and Threlfall, 1999). In this direction pattern can be seen the first step for generalization whereas generalization is the heart of algebra (Jones, 1993; Hargreaves, Shorrocks-Taylor and Threlfall, 1998).

Generalization of patterns improves algebraic thinking and constructs the concepts of variable and function (Lesley and Freiman, 2004). Besides generalization helps students to understand symbolic representations and interrelate among previous knowledge of arithmetic (Lannin, 2005, p.233). Thus it facilitates to proceed from arithmetic to formal algebra. Patterns tasks enable to observe and verbalize individuals` own generalizations and translate them symbolically (English and Warren, 1998). Due to that searching for patterns is a fundamental step in order to make generalization it is seen as a way of approaching algebra (Mason, Johnston-Wilder and Graham, 2005; Orton & Orton, 1999; Zazkis and Liljedahl, 2002). Therefore they are used by many educators as a pre-algebraic activity (Mason, 1996). In short, patterns have an important role as a bridge between generalization and algebra in primary level for providing constitution of algebraic thinking that is the base of formal algebra.

Formulation of the relationships and generalizations which are in the subcategory of patterns has recently taken an important place in curriculum of ABD and United Kingdom (NCTM, 2000; Zaskis and Liljedahl, 2002). Armstrong (2000) emphasizes that introducing and searching out the patterns at early grades develop algebraic thinking whereas Tall (1992) points out that proceeding to algebra is easier through generalization of arithmetic. Therefore, students need to have experiences related to patterns as from preschool education and teaching needs to include tasks which are oriented the figurative and numerical understanding of generalization (Rivera and Becker, 2005). Smith (2003) supports integration of them in programmes of instruction by emphasizing the relationship among patterns, functions, generalization and algebra.

Many researches emphasize the importance of patterns in mathematics (Risnick et al, 1987; Rawson, 1993; Zazkis & Liljedahl, 2002; Lannin, 2005; Radford, 2006; Becker & Rivera, 2006; Papic, 2007; Carraher et al, 2008; Amit and Neria, 2008; Mulligan et al, 2008) and present different strategies for generalizations of patterns. Examining students` generalization strategies of patterns is important in terms of learning advanced algebra. In addition, although many studies related to patterns and

IJGE

ISSN: 2146-9296
www.ijge.net

International Journal of Global Education-2013 *volume 2, issue 4*

generalizations of patterns have been conducted in international literature there is limited number of researches in Turkey due to that the subject of patterns incorporated into primary mathematics curriculum in 2005 (Tanışlı and Köse, 2011). However, getting knowledge about how students construct patterns, use cognitive processes and think in this construction is important for development of algebraic thinking. Thus there is need to conduct more studies related to patterns and generalization of them. On the other hand, RAND Mathematics Study Panel (2003) emphasizes, "because most studies have focused on algebra at the high school level, we know little about younger students' learning of algebraic ideas and skills" (p. 48). Therefore, the study aims to investigate strategies involved in generalizations of patterns of students attending primary level and gain some insight on their thinking processes. Hence, the study attempted to answers the following questions:

1. What are 7th and 8th grade students` generalization strategies of patterns?
2. How do students continue the patterns to a near and a far step?

Method

A qualitative research approach was chosen as methodology for this study since it best answers the questions what and how. This was a case study which provides to a depth understanding of the cases or a comparison of several cases (Stake, 1995). It was applied because the aim of the study was to obtain rich, detailed information reflecting the viewpoints of participants on how to figure out and generalize the patterns (Bogdan and Biklen, 1998, Glesne, 1999).

2.1.Study Group

This research was carried out total 8 students attending the teaching program at 7th and 8th grades in a primary school in İstanbul in the term of 2012-2013. Of these participants, 4 were at 7th grade and 4 were 8th grade. The students were selected by the teacher considering achievement profiles as well as volunteering and having good communication skills so that they were representative of the grade 7 and 8 classes.

2.2.Data Collection Tools

Open ended 4 problems related to patterns were used as data collection tool. The questions were composed of linear patterns problems which were chosen from textbooks. Curriculum of MEB related to 6-8. grades and the researches in our country show that linear and quadratic patterns are more included in the form of number sequence and visual. Therefore two of pattern problems were chosen to represent number sequence form and the other two were visual. However, linear patterns were chosen because they were more appropriate to be practiced by the students attending the second primary stage. Besides, three experts` opinions were taken in order to determine whether the problems were convenient in terms of aims so that the validity of language, grade and content was provided.

Semi-structured interviews were made with the aim of determining the pattern generalization strategies of students and exposing how students think while solving these problems. The fundamental aim of interviews is to determine cognitive abilities of individual and explore the richness in the opinions by revealing the concepts which individual has and the relationships among these concepts (Zazkis and Hazzan, 1999). Each student was interviewed in approximately 30 minutes. During interviews, students were asked to solve four pattern problems and explain what their answers were and how they found them. In addition they were expected to answer the explanatory questions such as “How did you think?”, “How did you solve?”, “Why?”. The students were videotaped while solving questions to record everything they knew and to analyze in detail. Using video as a research tool allows one to visit the situation repeatedly and to determine what the students have been thinking or what understanding has taken place (Borgen and Manu, 2002).

2.3.Analysis of the Data

The collected data was classified according to Table 1 which was constituted by (Akkan and Çakıroğlu, 2012) considering the patterns generalization strategies in the previous researches in the related literature (Amit and Neri, 2008; Ebersbach and Wilkening, 2007; Garcia-Cruz and Martion, 1997; Krebs, 2003; Lanin, 2003, 2005; Lannin, Barker and Townsend, 2006; Ley, 2005; Orton and Orton, 1999; Rivera and Becker, 2005; Stacey,1989; Swafford and Langrall, 2000; Steele and Johaning, 2004). Besides, according to Yıldırım and Şimşek (2008) the data must be described in detail and include direct quotations in order to provide the reliability and validity of the findings. Thus, direct quotations from dialogs were used to present a descriptive and realistic picture. While presenting the qualitative data to describe the researcher and students names such as S1-7, S2-7, S3-7, S4-7, S5-8, S6-8, S7-8, S8-8 and R were used (S1-7 represents first student and 7^{th} grade student whereas S5-8 means fifth student and 8^{th} grade student).

Table 1. Generalization Strategies of Patterns

Strategies	Properties
Counting	It includes calculating the number of components which form a shape or constructing a model or drawing a picture which describes the situation in order to calculate desirable qualification.
Recursive and Additive	It includes use of previous term in the pattern to find next term or terms. Students usually try to find difference between two terms and add obtained difference on last term in order to find next term.
Multiplying with Difference	It involves multiplying with the difference between two terms. It is generally encountered in the generalization of linear relationships. Students notice constant difference between terms and express n. term as multiplication of n and difference.
Whole-Object or Proportion	It includes use of proportional reasoning in solving of pattern problems. According to Lannin (2003) this strategy is use of a piece as a unit to construct a larger unit by using multiples of units.
Guess and Check	It involves rule estimation without paying attention whether the rule works or not. Student introduces an algebraic relationship (rule) representing the problem situation and doesn’t take note of validity of his rule during process. Algebraic construction generally consists of numbers and operations related to problem situation.
Contextual	It includes constructing a rule or formula focused on context, namely, information related to situation.
Explicit	It involves the generalization of relationship between two variables in order to determine any value. Since this strategy provides to determine the functions by using equations and formulas it can be used to find near and far terms. Therefore it enables to obtain n. term and write general rule.

Findings

The Pattern Generalization Strategies of Students

Question 1: Write the algebraic expression corresponding to 1, 5, 9, 13,… number pattern by using parameter. Find the number at the 13. step of pattern.

The students S2-7 and S3-7 try to wrote some general formulas without focusing on the relationship between term and the value of term and checked the accuracy of formulas by comparing the number and value of the step. However, they weren't able to find general formula. Although they couldn't express algebraically they applied *Guess and Check* strategy.

S2-7: Here, 1, 5, 9, 13… it increases four by four? ımmm…[he works on the question but isn't able to find the formula]… Unfortunately…[he continues to thinks]. 2n+3… [he thinks loudly and tries to understand giving numbers instead of n]. 2n+3, two times one is two, two plus three is 5. It proves.
R: What should you find in return for first term when you write one instead of n?
S2-7: One…ımmm. 4n+1, four times one is four plus one is five. It gives five.
R: But you said that you should find one.
S2-7: [he tried to obtain formula but wasn't able to find it].

S3-7: [she tries to solve question and think for a while]. Unfortunately…[she writes 2n+(n-1) on the paper]
R: What did you try to do in here? [points n.2+(n-1)]
S3-7: I thought according to five. Two times two is four, two minus one is one, if we add one on four, five comes. However, when I made the same for nine, three times two is six, three minus one is two, six plus two is eight. Because of that nine didn't come, it isn't suitable.

The students S1-7, S4-7 and S8-8 estimate approximate formula by focusing on the difference between terms and then check the accuracy of it for first few terms. So they obtained the general formula of the patterns using *guess and check* strategy.

S1-7: I tried n+4 but it didn`t work.
R: Ok if we explain the relationship, what kind of relationship is there?
S1-7: It continues increasing four by four.
R: If we want to express by using parameters what can we say?
S1-7: I guess I found… 4n-3. [thinks for a while and writes something]. Iımmm… for example when we checked with one, four times one is four, minus three is one, when we checked with two four times two eight, minus three is five… it proves the pattern.

S4-7: n+4… one plus four is five… it is true.

R: Is it first term?

S4-7: Yes it is.

R: For one what is the first term of pattern?

S4-7: Hmm. Ok, one. [he holds to think for a while]. 4n-3, four times one is four minus three is one. For first term, one times four is four, four minus three is one. Here is one [he shows the first term in the question]. For second term, two times four is eight, eight minus three is five. It proves the pattern.

The student S5-8 realizes the relationship between terms and using this relationship he expresses the general formula of pattern. In addition his thinking process shows that he has conceptual knowledge about the obtained formula. Here student use *Explicit* strategy while making generalization of pattern.

S5-8: Here, I first tried to find general term. They continue increasing four by four. So I write 4n. When we write one, four times one is four then we reduce three and first term becomes one. Thus rule is 4n-3.
R: Is 4n-3 general term?
S5-8: Yes.
R: Can you explain your thinking process?
S5-8: The pattern increases four by four so I write n near four. Then if it is 4n, in order to obtain one, namely, first term we should reduce three from 4n. Hence general term is 4n-3.

While the student of S6-8 was solving the linear number sequence pattern, she expressed that she learned a rule to solve this kind of problems. She applied *contextual* strategy due to that S6-8 tried to solve the problem by using a rule which includes letter symbols instead of numerical value. Although she tried to make generalization through rote rule her generalization was correct. In addition the student S7-8 tried to construct a rule focusing on the context and relationships and check the accuracy of it to be sure so she used contextual strategy as well.

S6-8: I solve the question in two ways. In the first, I find 13. term by counting but it is little hard because if 100. term was asked I couldn't count one by one. In mathematics lesson teacher showed a rule like a+(n-1)k. Here, a represents first term and k means common difference. In this pattern, common difference is four and first term is one. When we put them into the rule, 4n-3 is obtained.
R: If you don't know the rule, how do you find general formula? Is there any different way?
S6-8: I try to understand the relationship among terms.

S7-8: First, I tried to understand how the numbers increase and find that the difference is four. So I write 4n then I put one instead of n to find first term. Four times one is four and I reduce three to get one since first term is one. I check general formula for second term, four times two is eight then I subtract three from eight and obtain five. I confirm the accuracy of formula by checking for first few terms.
R: How did you decide to reduce three from 4n?

IJGE

ISSN: 2146-9296
www.ijge.net

International Journal of Global Education-2013 *volume 2, issue 4*

S7-8: I wrote the numbers in the formula. In order to find first term I had to write one instead of n so four times one is four. However, here we want to obtain one so we subtract three to find one.

Question 2: Write the rule of pattern given at the below table.

Sequence Number in Pattern	Stick Number
1	4
2	7
3	10
…	…
n	

The students S1-7, S2-7, S4-7 and S5-8 correctly find the general formula of pattern considering the difference and focusing on the relationship between terms and the place of term. In addition they explain the reasons of their thinking ways so it is seen that they have conceptual knowledge about generalization of patterns. Students solve the question by applying *Explicit* strategy.

S1-7: n.3+1. For one, one times three plus one is equal to four, two times three plus one is seven, three times three plus one is ten… like that.
R: How did you decide to write 3.n?
S1-7: Since these increased three by three [shows the right part of column].
R: How did you decide to add one?
S1-7: Namely, it was suitable [her statement wasn't clear while saying, she smiled].
R: Why was it suitable?
S1-7: In my mind, I multiplied three by one, it is three but here writes four so I had to find something else. I added one and it matched with the others.

S4-7: 3n+1. Three times one is three, three plus one is four. Three times two plus one is seven. It is true.

R: How did you decide to write 3n+1?

S4-7: It increases three by three so I used 3n and when I wrote one instead of n three comes to get four I added one.

The student S6-8 and S7-8 tried to solve the question using a rule that they learned in the mathematics lessons. It shows that they prefer not reasoning and their conceptual knowledge is restricted.

Since they used a rule that they have already known they apply *contextual* strategy. Although they tried to make generalization through rote rule the generalizations were correct. Besides since S3-7 use numeric calculation and focus on the relationships between numbers, namely, context in the patterns this student applies contextual strategy.

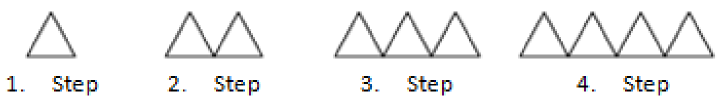
S7-8: There are the numbers of sequence and sticks in the table. The difference between terms is three. I use the rule in the form of a+(n-1)k. I write the difference and first stick number in the formula and find the general term as 3n+1.
R: From where do you know the rule?
S7-8: In a mathematics course, the teacher taught this rule.

S3-7: I found n.4 - (n-1). One times four is equal to four, one minus one is equal to zero, four minus zero is equal to four. For two, two times four is equal to eight, two minus one is equal to one, eight minus one is seven. It is suitable for all.
R: How did you decide to write n.4 - (n-1)?
S3-7: I looked at one and four, now… if we say four times... first I considered as four times. One times four is four, two times four is eight, three times four is twelve. I understood that I should reduce something from them. So, (n-1) one minus one is zero, if I get out of zero from four nothing changes. So I made in this way. I found for first. For second I had to reduce one, for third I had to reduce two. It was true for all.

The student S8-8 realized the increment value, however, although he wrote and checked many generalization formulas he weren't able to express the general term algebraically in written or verbally. This situation shows that the student use *Guess and Check* strategy.

S8-8: 4,7,10.. It increases three by three. [he writes some formula and check them..]. If you want next term, it is thirteen.
R: Ok. Can you express general formula?
S8-8: General formula.. immm..[he writes formulas such as n+1, 2n+1 but can`t find the formula].

Question 3:



a. Complete the 5. and 6. steps of the stick pattern.
b. Find the number of sticks is needed for 20.
c. Find the general formula of the pattern.

The students S1-7, S2-7, S3-7, S4-7, S5-8 and S7-8 correctly find the general formula of pattern considering the difference and focusing on the relationship between terms and the place of term. In addition they explain the reasons of their thinking ways so it is seen that they have conceptual knowledge about generalization of patterns. Students solve the question by applying *Explicit* strategy.

S1-7: There is one triangle in first step, two in second step… it is same with steps so there are respectively one, two, three, four, five and six triangles in first sixth step. Triangle composes of three sticks. So five times three is fifteen in 5. step and six times three is eighteen in the 6. step. I multiply step number by three so the general formula is found as 3n.

S4-7: Due to that there is one triangle in first step, two in second step and three triangle in third step, I understand there will be five triangles in fifth step and six triangles in sixth step
R: What is the general formula of patter?
S4-7: 3n.
R: why?
S4-7: Due to that triangle has three edges and ıımmm…it continues to increase one by one in each step, namely, the number of the step and the number of the triangle at this step is same, we multiply n with three.

S5-8: First, I count the number of sticks. There are three sticks in the first step, then six and nine. The question asks to find 5. and 6. steps. At first I find the rule of this pattern.
R: How do you find?
S5-8: It goes by increasing three by three so I write 3n. When I write one instead of n it becomes three and I obtain first term. So the rule is 3n and to find 5. and 6. steps I respectively write five and six instead of n and obtain the numbers in these steps as fifteen and eighteen.

While the student of S6-8 was solving the linear number sequence pattern, she expressed that she learned a rule to solve this kind of problems. Due to that S6-8 tried to solve the problem by using a rule which includes letter symbols instead of numerical value and she previously knew she applied *contextual* strategy. Although she tried to make generalization through rote rule her generalization was correct.

S6-8: I use the rule of a+(n-1)k and since difference among the numbers of sticks and first term are three I write three instead of both a and k. From here general formula of the pattern comes as 3n. It increases three by three so I continue by counting and find the numbers as fifteen and eighteen for 5. and 6. steps. However, in order to find 20. step I use general formula and write twenty instead of n and twenty times three, sixty is obtained.

The student S8-8 focuses on the difference between terms and maintains the patterns by adding difference on previous terms. So it is seen that *addictive* strategy was used here. In addition, although the student recognized how to maintain pattern he wasn't able to obtain general formula.

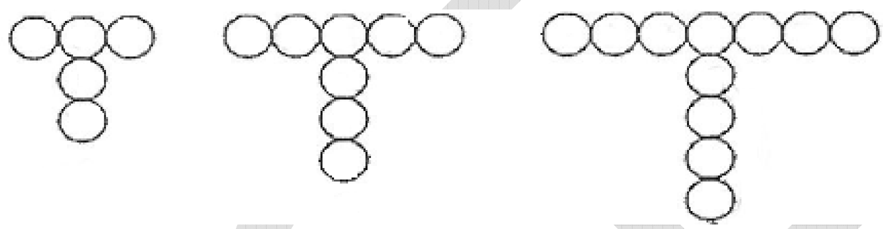
IJGE

ISSN: 2146-9296
www.ijge.net

International Journal of Global Education-2013 *volume 2, issue 4*

S8-8: I tackle the number of triangle. There is one triangle in first step, two triangles in second step and three triangles in third step. Since there are three sticks in each triangle first term is three, second is six, third is nine and the pattern goes by increasing three by three. Therefore it continues in the form of 12, 15, 18, 21. From here, it is seen that 5. term is fifteen and 6. term is eighteen.
R: Can you express general formula of pattern?
S8-8: It continue increasing three by three.. but iimm.. general formula [He thinks for a while but can`t find the rule of pattern].

Question 4:



a. Find the numbers of circles in the first five steps in.
b. Find the general formula of the pattern.

The students S3-7, S4-7, S6-8 and S7-8 tried to solve the question using a rule that they learned in the mathematics lessons. It shows that they prefer not reasoning and their conceptual knowledge is restricted. Since they used a rule that they have already known they apply *contextual* strategy. Although they tried to make generalization through rote rule S7-8 wasn't able to obtain general formula of pattern whereas S6-8 found the correct formula.

S7-8: [He counts the number of circles] it continues as 5, 8, 11. So the pattern increases three by three. When I use the rule a+(n-1)k a is the first term so I write five instead of it. The difference is three so I write it instead of k. When I write three instead of n I find eleven. Third term is eleven so general formula is true.
R: What is the general formula?
S7-8: a+(n-1)k.
R: Yes, it is a rule but what is the general formula of pattern?
S7-8: [He thinks on it] I couldn't find.

S4-7: I write 3n since pattern increases three by three. When I write one instead of n I obtained three but first term is five so I add two. Thus, I find general formula as 3n+2. Then I check for second and third steps by writing two and three in the formula and I see the accuracy of general formula. Then I write four and five in the formula and find the number of circles in 4. and 5. steps as fourteen and seventeen.

IJGE

ISSN: 2146-9296
www.ijge.net

International Journal of Global Education-2013 *volume 2, issue 4*

The student S8-8 focusing on the relationship between term and the place of it estimate approximate formula and then check the accuracy of it for first few terms. S5-8 obtained the general formula of the pattern but S8-8 wasn't able to get formula using *guess and check* strategy.

S8-8: The pattern goes increasing three by three so I continue adding three on previous term and find circles numbers in 4. and 5. steps. For next option..[He writes some formulas but can`t find correct one]. I couldn't find it.

The students S1-7, S2-7, and S5-8 correctly find the general formula of pattern considering the difference and focusing on the relationship between terms and the place of term. Students solve the question by applying *Explicit* strategy.

S1-7: The pattern goes increasing three by three so I write 3n. When I give one instead of n it comes three but I must add two on it to obtain five which is the first term of pattern. Thus the general formula of pattern is obtained as 3n+2. Then I write four and five in the formula and find the number of circles in 4. and 5. steps as fourteen and seventeen.

Table 2. The Pattern Generalization Strategies of 7^{th} and 8^{th} Grade Students

Students		The Type of Patterns			
		Number Sequence	Number Sequence	Visual	Visual
7^{th} Grade	S1-7	Guess and Check	Explicit	Explicit	Explicit
	S2-7	Guess and Check	Explicit	Explicit	Explicit
	S3-7	Guess and Check	Contextual	Explicit	Contextual
	S4-7	Guess and Check	Explicit	Explicit	Contextual
8^{th} Grade	S5-8	Explicit	Explicit	Explicit	Explicit
	S6-8	Contextual	Contextual	Contextual	Contextual
	S7-8	Contextual	Contextual	Explicit	Contextual
	S8-8	Guess and Check	Guess and Check	Additive	Guess and Check

Furthering the Pattern on Near and Far Steps

Some students focus on the difference between terms and find the next term by adding the difference on previous term. Here 5. and 6. terms are near steps and the students prefer *additive* strategy to find them.

S6-8: There are respectively five, eight and eleven circles in the first third step so I understand that it continues by increasing three by three. I find the circle numbers in 5. step by adding three by three.

S8-8: I tackle the number of triangle. There is one triangle in first step, two triangles in second step and three triangles in third step. Since there are three sticks in each triangle first term is three, second is six, third is nine and the pattern goes by increasing three by three. Therefore it continues in the form of 12, 15, 18, 21. From here, it is seen that 5. term is fifteen and 6. term is eighteen.

13. and 20. steps can be considered as near or far steps depending on the structure of the pattern. In terms of the questions that we asked in this study both steps can be calculated through a basic strategy like addictive or an advance strategy like explicit. The examples of dialogs show that some students first prefer obtaining the general formula and then find any asked term by using it. So we understand that some students use *explicit* strategy to find terms.

R: What is the number in 13. step?
S7-8: When I write 13 instead of n in formula, four times thirteen minus three is forty nine.

R: What is the number of sticks at the 20. step?
S3-7: Sixty
R: How did you solve this question?
S3-7: I multiply twenty with three since triangle has three edges.
R: Can we find the outcome in a different way?
S3-7: We can find drawing figures but 80. step may be asked to find and we can`t draw until 80. step. Thus, solving is easier by finding a formula.

R: What is the number of sticks at the 20. step?
S4-7: Twenty times three.
R: How did you decide to do like this?
S4-7: The number of edges of triangle is three. There are twenty triangles at 20.step so I multiplied twenty with three.

Although 5. and 6. steps are near steps the dialogs show that some students prefer making generalization and get general formula then they use it to find the asked steps. Therefore it is seen that some students use *explicit* strategy to find near steps.

S5-8: First, I count the number of sticks. There are three sticks in the first step, then six and nine. The question asks to find 5. and 6. steps. At first I find the rule of this pattern.
R: How do you find?
S5-8: It goes by increasing three by three so I write 3n. When I write one instead of n it becomes three and I obtain first term. So the rule is 3n and to find 5. and 6. steps I respectively write five and six instead of n and obtain the numbers in these steps as fifteen and eighteen.

S1-7: Thus the general formula of pattern is obtained as 3n+2. Then I write four and five in the formula and find the number of circles in 4. and 5. steps as fourteen and seventeen.

The following sample dialogs show that some students are in tendency to use *addictive* strategy to obtain near steps whereas they prefer finding the general formula and use *explicit* strategy to get far steps.

R: Ok, the question asks to find the number at the 13. step. Can you find that?
S2-7: forty nine.
R: How did you find it?
S2-7: I did it by adding four
R: If the question asks the number at the 120. step?
S2-7: I try to solve with formula… ıımmm…I did like this because of that the step number was small.

S6-8: It increases three by three so I continue by adding and find the numbers as fifteen and eighteen for 5. and 6. steps. However, in order to find 20. step I use general formula and write twenty instead of n and twenty times three, sixty is obtained.

Discussion and Conclusion

The results of this study show that 7th and 8th grade students` pattern generalization strategies are diverse. They are generally in tendency to use contextual or explicit strategies. According to the study of Akkan and Çakıroğlu (2012) 7th and 8th grade students mainly use contextual and explicit strategies in linear patterns so it supports our study in terms of used strategies. It shows that some students applied contextual strategy benefiting from a rule without paying attention conceptual understanding of the patterns or the relationships among terms. These students focused on obtaining the general formula and finding the value of asked steps. In addition some students used guess and check strategy because of not realizing the relationships between terms and not being sure about the accuracy of general formula. On the other hand, only one student preferred using addictive strategy since he wasn`t able to find general formula of pattern. In this study students usually tried to get general formula at first since they believe that answering any question by using general formula is easier. However, if they didn't find it they applied adding the difference between terms on previous term or counting the numbers in sequence.

In number sequence patterns six of students preferred using guess and check strategy whereas five of them used explicit strategy. Besides, five students applied contextual strategy for generalization. Some of the students who used guess and check strategy weren't able to discover the general rule of pattern or make generalization whereas some of them were enough to find general formula or express correctly the relationship between terms in algebraic form. It shows that there are deficiencies in students` structures of knowledge since they think a formula without being sure and need to check it or use a rote rule.

In visual patterns nine of the students preferred using explicit strategy whereas five of them used contextual strategy. In addition only one student used addictive strategy and guess and check strategy was used by one student for generalization as well. In this type of patterns students are in tendency to change visual patterns into number sequence (Becker ve Rivera, 2006; Krebs, 2005; Lan Ma, 2007; Orton ve

IJGE

ISSN: 2146-9296
www.ijge.net

International Journal of Global Education-2013 *volume 2, issue 4*

Orton, 1999; Stacey, 1989) and work with numbers so they adopt numerical approach that they find numeric equivalent of shapes in each step (Becker ve Rivera, 2005, 2006; Garcia-Cruz ve Martinon, 1997; Krebs, 2005; Lan Ma, 2007; Orton ve Orton, 1999; Orton et al., 1999; Stacey, 1989).

There aren`t too many differences between generalization strategies which are used in number sequence and visual patterns since they solved visual patterns by changing into number sequence. According to the research of Akkan and Çakıroğlu (2012) students are more successful in solving number sequence patterns than visual patterns so this finding doesn't show parallelism with our study. In this study, most of the students achieved to recognize the relationships between terms and obtain the general formulas of the patterns. However, some of them weren't able to make generalization. Many researchers indicate that students more successful in the pattern problems which they are familiar (Feifei, 2005; Lannin, 2005; Orton and Orton, 1999). Due to that 7^{th} and 8^{th} grade students are familiar to linear patterns and this type is appropriate in terms of students` cognitive levels most of students might be successful in making correct generalization in this study.

This study shows that students may prefer addictive or explicit strategy to find near steps. There are some students who used explicit strategy to find near steps whereas some students used addictive strategy. It can be said if students obtain general formula of patterns they can apply both strategy according to their preferences but if they aren't able to find formula they are in tendency to use addictive strategy to find near steps. On the other hand, they need to make generalization and obtain the general formula, they don't use addictive strategy for far steps since they know it is difficult. Due to that they have the opportunity to find the near steps by adding the difference between terms on previous term or counting the numbers in sequence, students are more successful in finding near steps in contrast with far steps. The finding is parallel with the finding of the study of Akkan and Çakıroğlu (2012). In generally, the students are in tendency to use *addictive* strategy to obtain near steps whereas they prefer finding the general formula and use *explicit* strategy to get far steps. Stacey (1989) and Tanışlı and Özdaş (2009) also support that the focus is on explicit strategies within the near generalization and on addictive strategies within near generalizations.

Some students aren't succeed in making correct generalization whereas some of them are tendency to use rote rule and haven't enough conceptual understanding. So teachers should use different type of patterns and solution strategies to make rich their perceptions and also focus on making students understand the relationships between term and the place of it in order to develop their algebraic thinking and constitute a substructure for advanced algebra.

References

Akkan Y. and Çakıroğlu Ü. (2012). Doğrusal ve İkinci Dereceden Örüntüleri Genelleştirme Stratejileri: 6-8. Sınıf Öğrencilerinin Karşılaştırılması. Eğitim ve Bilim, 37 (165), 184-194.

Amit, M. and Neria, D. (2008). Rising to the challenge: Using generalization in pattern problems to unearth the algebraic skills of talented pre-algebra students. *ZDM Mathematics Education*, 40, 111-129.

Barbosa, A., Vale, I and Palhares, P. (2012). Pattern Tasks: Thinking Processes Used by 6th Grade Students. Revista Latino *americana de Investigación en Matematica Educativa*, *15* (3) : 73- 293.

Becker, J. R., & Rivera, F. (2006). Sixth graders' fi gural and numerical strategies for generalizing patterns in algebra (1). In S. Alatorre, J. L. Cortina, M. Saiz, & A. Mendez (Eds.), *Proceeding of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 95-101). Merida, Mexico: Universidad Pedagogica Nacional.

Bogdan, R. C., & Biklen, S. K. (1998). *Qualitative research for education: An ıntroduction to theory and methods* (3rd ed.). Boston, MA: Allyn and Bacon.

Borgen, K. L., & Manu, S. S. (2002). What do students really understand? *Journal of Mathematical Behavior, 21*(2), 151-165.

Burns, M. (2000). *About teaching mathematics. A-K 8 research* (2nd ed.) Sausaluto, California. CA: Math Solutions Publication.

Carraher, D. W., Martinez, M. V., & Schliemann, A. D. (2008). Early algebra and mathematical generalization. *ZDM Mathematics Education*, *40*, 3-22.

Ebersbach, M. and Wilkening, F. (2007). Children's intuitive mathematics: The development of knowledge about nonlinear growth. *Children Development*, 78, 296-308.

English, L., D. and Warren, E. A. (1998). Introducing the variable through pattern exploration. *Mathematics Teacher,* 912, 166–170.

Dörfler, W.: 1991, 'Forms and means of generalization in mathematics', in A.J. Bishop (ed.), Mathematical Knowledge: Its Growth through Teaching, Kluwer Academic Publishers, Dordrecht, pp. 63–85.

Feifei, Y. (2005). "Diagnostic Assessment of Urban Middle School Student Learning of Pre algebra Patterns". Doctoral Dissertation, Ohio State University, USA.

Fox, J. (2005). Child-initiated mathematical patternning in the pre-compulsory years. In H. L. Chick & J. L. Vincent (Eds.), *Proceeding of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 313-320). Melbourne: PME.

Garcia-Cruz, J. A. and Martinon, A. (1997). Actions and Invariant Schemata in Linear Generalizing Problems. In E. Pehkonen (Ed.*) Proceedings of the 21th Conference of the International Group for the Psychology of Mathematics Education.* (pp. 289-296). University of Helsinki.

Goldenberg, E.P., Cuoco, A.A. and Mark, J. (1998). A role for geometry in general education. In R. Lehrer & D. Chazan (Eds.) *Designing Learning Environments for Developing Understanding of Geometry and Space.* Mahwah, NJ: Lawrence Erlbaum Associates, 3-44.

Guerrero, L., & Rivera A. (2002). Exploration of patterns and recursive functions. In D. S. Mewborn, P. Sztajn, D. Y. White, H. G. Heide, R. L. Bryant & K. Nooney (Eds.), *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (24th, Athens, Georgia, October 26-29)* (Vol. 1-4, pp. 262-272). Athens, Georgia: PME-NA.

Hargreaves, M., Shorrocks-Taylor, D., & Th relfall, J. (1998). Children's strategies with number patterns. *Educational Studies*, *24*(3), 315-331.

Hargreaves, M., Shorrocks-Taylor, D., & Th relfall, J. (1999). Children's strategies with number patterns. In A. Orton (Ed.), *Pattern in the teaching and learning of mathematics* (pp. 67-83). London and New York, NY: Cassell.

Herbert, K., & Brown, R. H. (1997). Patterns as tools for algebraic reasoning. *Teaching Children Mathematics*, *3*, 123-128.

Krebs, A. S. (2003). Middle grade students' algebraic understanding in a reform curriculum. *School Science and Mathematics*, 103, 233-243.

Krebs, A. S. (2005). Studying students' rea. *Mathematics Teaching in Th e Middle School*, *10*(6), 284-287.

Lan Ma, H. (2007). Th e potential of patterning activities to generalization. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), Proceeding of Th e 31th Conference of the international Group for the Psychology of Mathematics Education (Vol. 3, pp. 225- 232). Seoul: PME.

Lannin, J. (2003). Developing algebraic reasoning through generalization. *Mathematics Teaching in the Middle School*, 8(7), 342-348.

Lannin, J., Barker, D. and Townsend, B. (2006). Algebraic generalization strategies: factors influencing student strategy selection. *Mathematics Education Research Journal*, 18 (3), 3-28.

Lannin, J. K. (2005). Generalization and justification: The challenge of introducing algebraic reasoning through patterning activities. *Mathematical Thinking and Learning*, 73(7), 231- 258.

IJGE

ISSN: 2146-9296
www.ijge.net

International Journal of Global Education-2013 *volume 2, issue 4*

Ley, A., F. (2005). "A Cross- Sectional Investigation of Elementary School Students' Ability to Work with Linear Generalizing Patterns: The Impact of Format and Age on Accuracy and Atrategy Shoice". Master Dissertation,Toronto University, Canada.

Lesley, L., & Freiman, V. (2004). Tracking primary students' understanding of patterns. In M. J. Hoines, & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 415- 422). Bergen, Norway: PME.

Mason, J. (1996), Expressing Generality and Roots of Algebra. In N. Bednarz, C. Kieran and L. Lee (eds.), *Approaches to Algebra, Perspectives for Research and Teaching* (pp. 65–86). Dordrecht: Kluwer Academic Publishers.

Mason, J., Johnston-Wilder, S. and Graham, A. (2005). *Developing Thinking in Algebra*. London: Sage (Paul Chapman).

MEB, TTKB. (2006). *Ortaöğretim Matematik Dersi Öğretim Programı ve K*ılavuzu (ss.7,101) . Ankara: MEB Basımevi.

Mulligan, J. ; Mitchelmore, M.; Kemp, C. & Marston, J. (2008). Encouraging mathematical thinking through patterns and structure: An intervention in the first year of schooling. *Australian Primary Mathematics Classroom*, 13(8), 10-15.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA:NCTM.

Olkun, S. ve Toluk-Uçar, Z. (2006). *İlköğretimde matematik öğretimine çağdaş yaklaşımlar.* Ankara: Ekinoks Yayınları.

Orton, A. (1999). Pattern in the teaching and learning of mathematics. London: Cassel.

Orton, A. & Orton, J. (1999). Pattern and the Approach to Algebra. In A. Orton (Ed.) *Pattern in the Teaching and Learning of Mathematics* (pp. 104–120). Cassell, London.

Orton, J., Orton A. ve Roper T. (1999). Pictorial and practical contexts and the perception of pattern. In A. Orton (Ed.), *Pattern in the teaching and learning of mathematics* (121- 136). London and New York: Cassell.

Papic, M. (2007). Promoting repeating patterns with young children—more than just alternating colours! *Australian Primary Mathematics Classroom,* 12(3), 8-13.

Radford, L. (2006). Algebraic thinking and the generalization of patterns: A semiotic perspective. In S. Alatorre, J. L. Cortina, M. Saiz, & A. Mendez (Eds.). Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Vol. 1, pp. 2-21). Merida, Mexico: Universidad Pedagogica Nacional.

RAND Mathematics Study Panel. (2003). Mathematical Proficiency for all Students: Toward a Strategic Research and Development Program in Mathematics Education. Sta. Monica, CA: RAND.

Rawson, B. (1993). Searching for patterns. *Education 3-13, 21(3),* 26-33.

Reys, R. E., Suydam, M. N., Lindquist M. M., & Smith. N. L. (1998). Helping children learn mathematics (5th ed.). Boston, MA: Allyn and Bacon.

Risnick, L.B. Cauzinille-Marmeche, E. and Mathieu, J. (1987). Understanding algebra. In J. A. Sloboda and D. Rogers (ed), Cognitive processes in mathematics. Oxford: Clarendon Press.

Rivera, F. & Becker, J. (2005). Figural and numerical modes of generalizing in Algebra. *In Mathematics Teaching in the Middle* School, 11(4),198-203.

Smith, E. (2003). Stasis and Change: Integrating Patterns, Functions, and Algebra Throughout the K-12 Curriculum. In J. Kilpatrick, W. G. Martin, & D. Schifter (Ed.) *A Research Companion to Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Souviney, R. J. (1994). *Learning to teach mathematics* (2nd ed.). New York, NY: Merrill.

Stacey, K. (1989). Finding and using patterns in linear generalizing problems. *Educational Studies in Mathematics*, 20, 147–164.

Stake, R. (1995). *The art of case study research.* Thousand Oaks, CA: Sage

Steele, D. & Johanning D. I. (2004). A schematic–theoretic view of problem solving and development of algebraic thinking. *Educational Studies in Mathematics*, 57, 65–90.

Steen, L. A. (1988). The Science of Patterns, *Science*, 240, 611–616.

Swafford, J. O. & Langrall, C. W. (2000). Grade 6 students' pre-instructional use of equations to describe and represent problem situations. *Journal for Research in Mathematics Education, 31*(1), 89–112.

Tall, D. (1992). The Transition to Advanced Mathematical Thinking: Functions, Limits, Infinity and Proof. D. Grouws(Ed.) *Handbook of Research on Mathematics Teaching and Learning* (pp. 495-514). Macmillan Publishing Company, Newyork.

Tanışlı, D. and Özdas, A. (2009). The strategies of using the generalizing patterns of primary school 5Th Grade students. *Educational Sciences: Theory and Practice,* 9 (3), 1485-1497.

Tanışlı and Köse (2011). Lineer Şekil Örüntülerine İlişkin Genelleme Stratejileri: Görsel ve Sayısal İpuçlarının Etkisi. Eğitim ve Bilim, 36 (160), 184-194.

Waters, J. (2004). Mathematical patterning in early childhood settings. In I. Putt, R. Faragher ve M. McLean (Ed.), *Mathematics Education For The Third Millennium, Towards 2010, MERGA* (*Mathematics Education Research Group of Australasia Conference Proceedings).*http://www.merga.net.au/publication/conf-_display.php?year=2005 adresinden 12.11.2006 tarihinde alınmıştır.

Yıldırım, A. ve Şimşek, H. (2008). Sosyal bilimlerde nitel araştırma yöntemleri (beşinci baskı). Ankara: Seçkin Yayıncılık.

Zaskis, R. & Hazzan, O. (1999). Interviewing in mathematics education research: Choosing the questions. *Journal of Mathematical Behaviour*, 17 (4), 429-439.

Zazkis, R. & Liljedahl, P. (2002). Generalization of patterns: The tension between algebraic thinking and algebraic notation. *Educational Studies in Mathematics*, 49, 379-402.