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MISTAKE HANDLING LEARNING IN MATHEMATICS

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Abstract

This study examines how middle school eighth-grade students deal with mathematics errors in their learning and how gender influences that approach using descriptive correlation analysis. Seventy eighth-graders from a public middle school in Turkey participated in the study during the 2024–2025 school years. Pupils employed a learning scale that dealt with mistakes. Both descriptive and correlational analyses were carried out. The findings showed that students understood the value of making mistakes because they scored higher on the scale. Significant, positive, and strong relationships were found through correlational analyses between each mistake-handling sub-dimension and the total score. Notable correlations were also discovered between the students' fear of making mistakes and teachers' emotional and cognitive approaches to mistakes. Gender had no significant impact on the mistake-handling learning strategy. Making mistakes in mathematics is something that many students fear and avoid. However, every mistake made in mathematics class is a learning opportunity.

Keywords: Correlational research, error management, error handling learning, mathematics education, mistake handling learning.

INTRODUCTION

Being intelligent is required in the new global order. This implies that people who are adept at using their minds will be able to adjust to the new global order more easily. These mental abilities are known as higher-order thinking skills by NCTM (2000). Reasoning, problem-solving, problem-posing, searching, drawing conclusions and generalizations, providing evidence for why an answer is right or wrong, and other cognitive abilities are examples of higher-order thinking skills (Walle, Karp, and Bay-Williams, 2013). The five mathematical processes—problem-solving, reasoning and proof, communication, creating connections, and representation—all call for higher-order cognitive abilities, according to NCTM's (2000) standards.

Countries have adjusted their curricula to reflect the importance of higher-order thinking skills (MEB, 2018; Van den Heuvel-Panhuizen, 1996). The goal of these adjustments is to develop critical thinkers through methods of questioning, analysis, and synthesis rather than memorization. Analysis and questioning necessitate negative knowledge. Negative knowledge refers to what is wrong and where the lines are drawn between what is right and wrong (Akpınar and Akdoğan, 2010). Negative information prompts introspection, introspection prompts questioning, and questioning prompts higher-order cognitive processes (Heinz, 2005). Since only positive knowledge consists of repeating correct information, higher-order thinking skills require both positive and negative knowledge. Many things in life are easier to understand when their opposite is present. White, for instance, looks best against at a black background. Knowing all of a thing's features and how they differ, such as what is true and what is false, is crucial to understanding it completely. As a result, negative knowledge is necessary for positive knowledge. Positive and negative knowledge will soon be required to grow



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higher-order thinking abilities and adjust to a new environment. To optimize learning about handling mistakes, educators should arrange their lesson plans and set up their classrooms (Bray, 2011).

Negative knowledge is described by Heinze (2005) as being inconsistent with what is already known. Negative knowledge is also referred to as "what not to do on the way to the right," and Tauber (2009) emphasizes that negative knowledge is discovered via trial and error. Negative knowledge, according to Gartmeier (2009), is how one avoids a destination. Consequently, knowing the limits of what is right and wrong while conducting transactions with a specific goal could be considered negative knowledge. It is crucial for the capacity to think at the level of wisdom because inquiry cannot be conducted without negative information (Akpinar and Akdoğan, 2010). Minsky (2004) asserts that negative knowledge is essential to the completion of the thought process and comprises a variety of thinking styles. Positive knowledge alone in education stifles inquiry because it discourages people from taking chances and exploring novel directions (Karadağ, 2009). Reflective thinking is triggered by cognitive conflict, which is brought on by negative knowledge. Furthermore, students begin to think and question more when they receive appropriate guidance after making mistakes and reflecting on them (Karadağ, 2009). As a result, the teaching process must include both positive and negative knowledge.

According to Parvianien and Eriksson (2006), there are three aspects of negative knowledge: understanding what we don't know, understanding what we don't do, and appreciating failure. Negative knowledge can be defined as the understanding of three things: what something is not, how something is not working, and strategies that are not appropriate for resolving the issue (Gartmeier, Gruber, & Heid, 2007). Four aspects of negative knowledge were examined by Lambe (2006): failures, framing knowledge, knowing what one does not know, and knowing what not to do. Negative knowledge boosts productivity, inquiry and reflective thinking, personal growth, and problem-solving skills (Akpınar and Akdoğan, 2010).

Negative information is often mistaken for inaccurate or misleading responses, and since it is accepted as the standard, negative feelings are attached to it. It goes beyond these, though. Constructivism and metacognition both make use of negative knowledge (Parviainen and Eriksson, 2006). Negative knowledge, then, informs people about boundaries and is personal, acquired via experience and trial and error. Possessing both positive and negative knowledge allows an individual to evaluate themselves realistically and attain mastery of learning.

Heinz (2005) developed the concept of mistake-handling learning—a method of effectively utilizing negative knowledge—because it is crucial to the teaching and learning processes. Turning mistakes into learning opportunities and integrating positive and negative knowledge into teaching and learning processes are key components of mistake-handling learning (Heinz, 2005). According to Karadağ (2009), mistake-handling learning involves allowing students to make mistakes and helping them learn from them. Error-handling learning is a self-directed learning approach that builds knowledge from errors (Heinz, 2005; Karadağ, 2009). Heinz (2005) asserts that while mistakes made by the majority of the class should be taken into consideration during the class discussion, individual mistakes should be addressed by the person in question. It is believed that emphasizing mistake-handling learning will lead to more effective learning (Borasi, 1994; VanLehn, 1999; Siegler, 2002). The learning process for mistake handling is depicted in Figure 1. The mistake is first detected. Then, a class discussion, student-teacher collaboration, or student-only inquiry is conducted using a highly reflective thinking approach. This process both corrects and prevents errors.

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Figure 1. Learning in mistake situations (Rach, Ufer and Heinze, 2013, p.23)

A recent concern in the literature on mathematics education is mistake-handling learning. Numerous studies have been conducted on students' mathematical errors, but few have examined the effects of errors on the learning process (Heinz, 2005). Consequently, research on error-handling learning is highly beneficial to the literature on mathematics education.

Heinz (2005) breaks down mistake-handling learning into four sub-dimensions: Students' individual evaluation of mistakes, teacher's emotional approach to mistakes, teacher's cognitive approach to mistakes and students' fear of making mistakes. Individual error management involves an individual's assessment of their errors as well as their fear of making mistakes (Heinz, 2005). According to Heinze (2005), an educator's affective approach to mistakes is referred to as their emotional approach, while their cognitive approach to mistakes is defined as how they intervene with mistakes.

There are some related studies. The study conducted by Aksu, Özkaya, Gedik, and Konyalıoğlu (2016) sought to examine the connection between mathematics anxiety, mistake-handling learning awareness, and mathematics self-efficacy. 323 pupils in the seventh grade took part in the study. The Self-Efficacy Scale, the Mathematical Anxiety Scale, and the Mistake-Handling Learning Scale were used to gather data. It was discovered that mathematical anxieties, mathematical self-efficacy, and mistake-handling learning were significantly and positively correlated. Furthermore, it was found that 51% of the variance in mathematical anxiety was explained by mistake-handling learning and mathematical self-efficacy.

The study conducted by Heemsoth and Heinze (2014) sought to determine how learning fractions is impacted by mistake-handling learning. 195 sixth-grade students participated in a quasi-experimental design that included pre-and post-tests. It took three weeks to complete the experiment. Throughout the study, the control group received traditional instruction on the subject while the experimental group learned through activities involving handling mistakes. Finally, it was discovered that advanced students gained more from mistake-handling activities and that incorrect examples increased students' negative knowledge.

Heemsoth and Heinz (2016) planned an experimental study to look into how students' achievement with fractions is impacted by learning how to handle mistakes. It took 2.5 weeks to complete the research. 174 students in the seventh and eighthh grades took part in the study. Students were given tests on fractions as pre-test, post-test, and follow-up. Students in the experimental group were instructed through error handling, which included reflective thinking about their mistakes and indepth discussions of the reasons behind their false answers, among other activities. In contrast,

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students in the control group were only taught the right answers when they made mistakes. Students in the experimental group significantly improved their procedural and conceptual knowledge in the post-test and follow-up exams.

55 primary education faculty students participated in an experimental study designed by Keith and Frese (2005) to examine the impact of error management on performance. Students in the experimental group were encouraged to make mistakes and grow from them, while students in the control group were instructed to emphasize an error-avoidant style. Their task was to use a computer to create a PowerPoint presentation. Students in the experimental group ultimately performed better than those in the control group.

To determine the impact of mistake-handling instruction on middle school students' attitudes toward errors and their mathematical performance, Rach, Ufer, and Heinz (2013) created a quasi-experimental study. 571 middle school students from grades 6-7-8 and 9 in Germany took part in the study. It took five months to complete the research. The experimental group analysed errors with proof because they were part of an error-tolerant culture. Students in the control group routinely gained knowledge from proofs. Students were given an achievement test and the Mistake-handling Learning Scale. Students in the experimental group performed better on the mistake-handling scale, but on achievement tests, there was no significant difference between the groups.

Research on mistake-handling learning has shown that it generally increases mathematical comprehension and reduces anxiety and other negative emotions related to the subject. A very limited study looking into middle school students' perceptions of handling mistakes in math learning was conducted. The current study looks into how eighth-grade students handle mistakes in their learning when they are in math classes. Thus, the following are the research questions:

1) How well do eighth-grade students handle mistakes in their mathematical learning strategies?

2) Is there a statistically significant relationship between the overall score and every sub-dimension of mistake-handling learning?

3) Does the gender of eighth-grade students affect how they approach learning mathematics and how they handle mistakes?

METHOD

The current study uses descriptive correlational research to find out how eighth-grade students approach mistake-handling learning in math classes and to investigate potential relationships between the gender effect and each sub-dimension of the mistake-handling learning approach.

Participants

Seventy 8th students from a public middle school in Nazilli/Aydın participated in the current study during the 2024–2025 academic years. There were 36 boys and 34 girls present. Every student shared an almost identical socioeconomic background and had comparable educational attainment from their parents.

Instruments

A 27-item, 4-point Likert scale for mistake handling was created by Spychiger, Mahler, Hascher, and Oser (1998). The scale was piloted by the researcher with 295 Swiss fourth- and ninth-graders. The scale's three primary components—teacher behaviour, personal feelings, and individual application of errors in error-handling scenarios—were revealed by factor analyses. Heinze (2005) mathematically adjusted the scale. Eighthy-five German students in the eighthh and ninth grades made up the sample. The four elements of the scale are the teacher's emotional approach to mistakes, their cognitive approach to mistakes, students' fear of making mistakes, and how each handles errors. The scale's sub-dimensions internal consistency coefficients varied from 74 to 88.

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The scale was translated into Turkish by Fidan, Doluzengin, Kasapsaraçoğlu, Sert and Muhammed (2017). The sample of the pilot research consists of 1229 secondary school students in the central districts of Denizli/Turkey. 50.9% of the participants were girls and 49.1% were boys; 34.8% are 6th grade students, 33.5% are 7th grade students, and 31.7% are 8th grade students. Data were analyzed using SPPS and AMOS 22 programs. Exploratory and confirmatory factor analyzes were used to test the construct validity of the scale. Cronbach Alpha reliability analysis was performed to test the reliability of the scale

Firstly, confirmatory factor analysis (CFA) was conducted to test the suitability of the 27-item, 4factor model proposed in the Mistake-handling Questionnaire adapted to mathematics by Heinze, to Turkish culture (N = 629). CFA results showed that the 27-item, 4-factor structure was not suitable for Turkish culture (X 2(318, N = 629) =937.520, p < .001; X 2/df =2.948). GFI = .89, RMSEA = .056 (.052-.060), SRMR = .079, CFI = .84, TLI = .82, IFI = .84). Then, exploratory factor analysis (EFA) was conducted to test the factor structure of the scale (N=629). As a result of EFA, it was seen that 5 items were loaded on factors other than the required factor. These items were removed from the scale and the 22-item scale was subjected to exploratory factor analysis again. Analysis results showed that it had a 4-factor structure that explained 48.22% of the total variance. The first factor consists of 8 items and accounts for 22.60% of the total variance, the second factor consists of 5 items and accounts for 11.83% of the total variance, the third factor consists of 6 items and accounts for 8.66% of the total variance, and the fourth factor consists of 3 items and accounts for 5.12% of the total variance explains. CFA was conducted again to test the suitability of this new 22-item structure for Turkish culture. The analysis results confirm the suitability of the 22-item, four-factor structure (X 2 (200, N = 600) = 442.783, p < .001; GFI = .94, RMSEA = .044 (.038-.049), SRMR = .060, CFI = .92, TLI = .91, IFI = .92).

Then, Cronbach Alpha reliability analyzes were conducted to provide evidence for the reliability of the scale. The analysis results show that the Cronbach Alpha reliability coefficients for the subdimensions of the scale are $\alpha = .73$ for the Reconsidering Errors sub-dimension, $\alpha = .73$ for the Teacher's Affective Behaviors in Error Situation sub-dimension, $\alpha = .74$ for the Afraid of Making Mistake sub-dimension, and $\alpha = .74$ for the Teacher's Cognitive Behavior in Error Situation sub-dimension. It showed that $\alpha = .83$ for the Behavior subscale.

In conclusion, the researchers examined the collected data. The Turkish version of the scale consists of 22 items with four components because four of the items were deemed inappropriate for Turkish culture. Moreover, the Turkish version uses a four-point Likert-type scale with consistency between its components ranging from 73 to 83. One could argue that the scale resembles Heinz's (2005) Turkish mistake-handing scale the closest.

The distribution of questions in the mistake-handling learning scale is displayed in Table 1. Eighth items make up the sub-dimension for the students' assessment of mistakes, six items represent the teacher's emotional approach to mistakes, three items represent the teacher's cognitive approach to mistakes, and five items represent the students' fear of making mistakes. The majority of the items deal with the subjective assessment of errors.

Sub-dimensions	Questions	Total	Percentage
Students' individual evaluation of mistakes	1,2,3,4,5,6,7,8	8	36,36%
Teacher's emotional approach to mistakes	9,10,11,12,13,14	6	27,27%
Teacher's cognitive approach to mistakes	15,16,17	3	13,63%
Students' fear of making mistakes	18,19,20,21,22	5	22,72%

Table 2 shows some examples of the mistake-handling learning scale.



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Table 2. Some examples of the mistake-handling learning scale

Sub-dimens	sions of	mistake-handl	ing	Scale items		
learning ap	proach					
Students'	individual	evaluation	of	* I think a lot about my erroneous solutions to math problems.		
mistakes				* I find the mistakes I made in the math class and try to understand my		
				mistakes.		
				* If I make a mistake in math class, I see my mistake as an opportunity to		
				learn.		
Teacher's	emotional	approach	to	* If someone makes a big mistake, our math teacher can't admit it and yells.		
mistakes				* I fear my math teacher when I make a lot of mistakes in written exams.		
				* Making mistakes is not a bad thing for our math teacher.		
Teacher's cognitive approach to mistakes		kes	* When I make a mistake in math class, my teacher talks to me about it in a			
				way that contributes to me.		
				* If I make a mistake in the math lesson, my teacher helps me learn the truth		
				about it based on the mistake.		
Students' fear of making mistakes				* I feel embarrassed if I make a mistake in front of the class in math class.		
		5		* Before the math lesson, I sometimes have the fear that I may make		
				mistakes in the lesson.		

Data Analyses

Data analyses were done using the SPSS 22.0 program. The normality test was done. The mean, maximum, and minimum scores were determined. Correlation analyses and non-parametric tests were done.

RESULTS

The Kolmogorov-Smirnov test was performed to determine whether the scores were normal because the sample size was 70, which was greater than 50 (Baykul and Güzeller, 2014). The findings of the Kolmogorov-Smirnov normality test are displayed in Table 3. The error-handling learning scale scores of eighth graders followed a normal distribution.

Table 3. Test of normality

Statistic	df	Significance	
,082	70	,200	

Calculations were made for descriptive analyses, which included mean scores, standard deviations, and minimum and maximum scores. The descriptive analyses of the eighth-grade students' errorhandling learning component scores are displayed in Table 4. The category with the highest mean score was individual mistakes evaluations. It follows that students are generally conscious of the value of mistakes and can assess them on their own.

Table 4. Descriptive statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Students' individual evaluation of	70	15,0	32,0	25,943	2,7917
mistakes					
Teacher's emotional approach	70	9,0	24,0	17,629	3,2176
Teacher's cognitive approach	70	7,0	12,0	10,629	1,3744
Students' fear of making mistakes	70	5,0	20,0	12,557	3,5494

Correlational analyses are displayed in Table 5. Parametric correlation analyses using Pearson Correlation Analysis were conducted because the scores exhibited a normal distribution. The following is a list of the findings:



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Table 5. Correlations

		Gender	Individual evaluation of mistakes	Teacher's emotional approach	Teacher's cognitive approach	Fear of Total making score mistakes
Gender	Correlation Coefficient	1,000				
Individual evaluation of mistakes	Significance Correlation Coefficient	-,070	1,000			
Teacher's emotional approach	Significance Correlation Coefficient	,565 ,203	-,093	1,000		
Teacher's cognitive approach	Significance Correlation Coefficient	,091 ,019	,446 ,226	,268*	1,000	
Fear of making mistakes	Significance Correlation Coefficient	,876 -,168	,060 -,121	,025 ,282*	-,062	1,000
Total score	Significance Correlation Coefficient	,165 -,013	,317 ,312**	,018 ,694**	,612 ,369**	,691**
* Correlation is	Significance	,917 05 lavel (2 tai	,009	,000	,002	,000 .

**. Correlation is significant at the 0.05 level (2-tailed).

Neither the overall score nor any of the sub-dimensions of the mistake-handling learning approach were significantly impacted by gender. Every sub-dimension of the mistake-handling learning approach showed strong, positive correlations with the overall score. The teacher's cognitive approach to mistakes and her/his emotional approach to them showed a strong and positive correlation. The teacher's emotional responses to errors and students' fear of making mistakes were significantly and positively correlated.

DISCUSSION and CONCLUSION

Learning from mistakes is an issue that should be emphasized not only in mathematics education but also in all field education. Because in the education approach of learning from mistakes, learning is not only focused on positive knowledge, that is, learning does not only focus on what is right, but also draws a comprehensive learning road map by focusing on what is not right, why and how, and which methods do not lead to the right result and why. Thus, it will be possible to learn the concept to be learned in a meaningful way, in all its aspects and based on the cause-effect relationship. This is exactly what is expected from modern individuals in the 21st century. It has become the common goal of almost all countries to raise individuals who produce and process information, and who can shed light on new ideas by transferring that information to different fields, rather than individuals who memorize information and who are inquisitive, critical, and able to make effective predictions and evaluations. Therefore, it is thought that learning from mistakes or error-based learning approach is very important in both mathematics education and the education of all fields.

This study uses descriptive correlation analysis to look at how eighth-grade students handle errors in their learning and how gender affects that process in math classes. It was discovered that each subdimension of mistake-handling learning and its overall score had a significant, favourable, and strong

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relationship. Furthermore, a strong correlation was observed between the emotional approach of a teacher and their cognitive approach to mathematics, as well as between their emotional approach and students' fear of making mistakes. Gender had no significant effect on learning how to handle mistakes.

Mathematical achievement is increased by mistake-handling learning approaches (Heemsoth and Heinze, 2014; Heemsoth and Heinze, 2016; Keith and Frese, 2005; Rach, Ufer, and Heinz, 2013). Furthermore, studies by Aksu, Özkaya, and Konyalıoğlu (2016) and Rach, Ufer, and Heinze (2013) demonstrated the relevance of the mistake-handling learning approach to affective features. These studies focused on how an environment that accepts mistakes in the classroom can help students turn them into teaching moments. Furthermore, the same results were confirmed by the current study. Thus, it is possible to conclude that learning how to handle errors improves math achievement and is associated with instructors' emotional and cognitive responses to errors as well as a classroom environment that is error-tolerant. Teachers ought to establish error-tolerant classrooms and recognize the importance of making mistakes.

In the light of the experiences gained as a result of this research, some ideas for new research have been generated. Teacher and student awareness should rise because mistake-handling learning boosts math achievement. The impact of a teacher's emotional and cognitive style or an error-tolerant culture on students' approach to learning how to handle mistakes could be the subject of experimental research. Teachers could attend seminars on the importance of making mistakes and how to turn them into teaching moments. It is predicted that as the educational approach based on learning from mistakes is adopted, students will grow up to be more self-confident, happier individuals, who love to learn, who are not afraid of trying new ideas, and who are more enterprising. Therefore, it is recommended that teachers teach with an approach that adopts an error-based education approach.

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