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INVESTIGATION OF THE EFFECT OF TECHNOLOGY SUPPORTED TEACHING OF QUADRATIC SURFACES ON THE CRITICAL THINKING SKILLS OF MATHEMATICS TEACHER UNDERGRADUATE STUDENTS[1]

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Abstract

It is known that within the scope of 21st century skills, the basic skills that individuals should have are using information and communication technologies effectively and having critical and reasoning skills. One of the most important purposes of the education we receive in our schools is to enable the individual to develop his thinking ability. Critical thinking skills are among the most important in teaching mathematics. Our study aims to examine the effect of technology-supported teaching on the critical thinking skills of mathematics teaching students, as technology has developed rapidly in recent years and become a part of our educational life. In our study, a mixed method in which quantitative and qualitative research was used together was chosen. Critical Thinking Disposition Scale was used to collect quantitative data; Quadratic Surfaces Real Life Problems Activity was used to collect qualitative data. While the subject was taught to the experimental group using Geogebra dynamic software, the traditional teaching method was used to the control group. While it was seen that using Geogebra after the application was more effective in positively changing the critical thinking tendency of the experimental group students. When the qualitative data were examined, it was observed that the students in the experimental group were much more successful in their critical thinking skills in solving the questions, especially in the inference explanation and evaluation steps.

Keywords: Critical thinking, **t**echnology, geogebra, quadratic surfaces.

INTRODUCTION

Thinking is the most important feature that distinguishes humans from other living creatures. The concept of thinking has been a subject of interest for many philosophers from past to present. Socrates "I can't teach anyone anything, I can only make them think." While Kantarcı (2013) states, Ömer Hayyam also says, "As long as I think, the world exists, I do not exist, and neither does it." (Eyüboğlu, 2003) and emphasized the importance of thinking.

According to the Turkish Language Association (TDK, 1992), thinking; It is defined as examining and comparing information in order to reach a conclusion and generating ideas by making use of the interconnected interests. According to Cüceloğlu (1996), thinking is an active, purposeful and organized mental process performed to understand the situation. Generally speaking, thinking includes one or more skills. Skills such as reasoning, decision making, critical thinking, creative thinking and problem solving are important for the thinking process to take place, starting from reading comprehension. Halpern (2014) defines critical thinking, one of the types of thinking, as "purposeful, logical and goal-

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oriented thinking that requires solving problems, determining inferences, calculating possibilities and making decisions." According to Paul (1992), critical thinking can be defined as not only thinking, but also thinking about what is effective in the individual's self-improvement, improving himself in terms of thinking by using patterns to evaluate thinking, and reaching conclusions based on observation and knowledge (Paul and Elder 2002). As a result of all these definitions, critical thinking; reaching a balanced judgment after examining previous experiences, knowledge and thoughts and evaluating different opinions, looking at an issue or event correctly and logically (Nosich, 2018), evaluating the reality and value of an event with certain standards, based on a well-grounded judgment It is stated as (Önal, 2020).

Critical thinking skills, evaluated as a process, consist of some components. Facione (1998) defines the basic components of critical thinking; These are analysis, interpretation , self-regulation, inference, explanation, and evaluation. These components,

> *Analysis:* Identifying actual relationships between situations, problems, concepts, definitions, or other kinds of representations designed to express beliefs, judgments, beliefs, knowledge, or opinions.
> *Interpretation:* Determining and understanding the meaning and significance of various experiences, situations, data, events, judgments, beliefs, rules, procedures or classifications.
> *Self-Regulation:* Controlling a person's cognitive activities and the components used in these cognitive activities and regulating the situation according to the results obtained.
> *Inference:* Identifying the components necessary to draw logical conclusions, shaping hypotheses and predictions, taking into account information on the subject; It is defined as reaching conclusions based on data, judgments, opinions, concepts and definitions.
> *Explanation:* Identifying a person's reasoning situation and process and presenting their reasoning in the form of convincing arguments.
> *Evaluation:* Evaluating the plausibility of statements or other representations that are accounts, accounts, or descriptions of a person's perception, experience, situation, judgment, belief, or opinion, and assessing the logical strength of fact or inferential relationships between statements, explanations, questions, or other forms of representation,

it is explained as.

In the information age we live in, where technology is rapidly developing, very rapid developments and changes are taking place in the world. Societies make great efforts to catch up with these developments and keep up with the changes. With the rapid development and change of science and technology, accessing, disseminating and learning information has become much easier (Demir, 2014). Due to the need to train individuals who can use this knowledge effectively in the right place, new approaches have been sought in education. For this reason, we have been faced with the need to redefine the goals and objectives of education in order to create individuals who meet the requirements and requirements of the time they live in (Maričića, Špijunović, & Lazić 2016).

In the studies carried out, 21st Century thinking skills are emphasized. These skills are stated as creativity, critical thinking, problem solving, decision making and learning (Yılmaz, 2016). Organizations such as the United Nations Educational, Scientific and Cultural Organization (UNESCO) and the European Council on Education have drawn attention to the increased activity of students in the learning process, with an emphasis on personal development, creativity, autonomy, the development of thinking and especially the development of critical thinking skills (Maričića, Špijunović, Lazić 2016). Technology can be used effectively in education to ensure the development of critical thinking skills (Wagner, 2008), which are among the 21st century skills. The National Educational Technology Standards (NETS) community in America has stated that educational technologies should be used to improve student learning, increase productivity and encourage creativity (Smaldino, Lowther, Russell

& Mims 2007). However, having computers and other technologies in the classroom is not enough. The important thing is that the teacher must be knowledgeable about applying this technology in the classroom.

Kokol-Voljc (2007) stated that one of the main issues in training future teachers, especially mathematics teachers, is to train them to use technology appropriately in their teaching. Karataş and Güven (2008) stated that teacher candidates should be taught how to use technology in mathematics education by giving example situations, and Clarke (2009) stated that teacher candidates should be trained to use technological skills in order to see the power of technology in increasing mathematics knowledge.
For these reasons, it is aimed to examine the effect of technology-supported teaching of quadratic surfaces on critical thinking of mathematics teaching undergraduate students, who are prospective teachers of the future.

METHOD

In our study, in which the acquisition of critical thinking skills of undergraduate mathematics teachers was evaluated, a mixed method was chosen, in which we used both quantitative and qualitative research methods. Mixed method research is a research method in which the research problem is examined in a multidimensional and comprehensive manner and quantitative and qualitative methods are used together (Yıldırım & Şimşek, 2003).

Research Design

It is planned to use an experimental model in the quantitative part. In our study, it was planned to use a pretest-posttest experimental model with a control group. In this model, one of the groups is determined impartially as the experimental group and the other as the control group. The researcher applies the pre-test to both groups, and the experimental procedure is applied only to the experimental group. The posttest is then applied to determine the difference between the two groups (Creswell 2021). Before starting the application, the Critical Thinking Disposition (CTD) Scale was applied as a pre-test to the students in the experimental and control groups. After the pre-tests were applied, technology-supported quadratic surfaces were taught to the students in the experimental group by the researcher, while in the control group, the lessons were taught with traditional teaching in accordance with the curriculum. After the teaching, the Critical Thinking Disposition (CTD) Scale was applied to the experimental and control group students as a posttest.

In the qualitative part, the case study method was chosen from qualitative research methods.Case study is a research method that is based on how and why questions and allows the researcher to examine in depth a phenomenon that he cannot control (Yıldırım and Şimşek, 2003). The aim of the case study is not to reach generalizations, but to reveal how and why results and opinions. Since this part of this research aims to examine critical thinking skills in depth, a case study was thought to be appropriate.
In our study, which we conducted as a case study, we aim to seek answers to why and how questions regarding critical thinking. Since the subunits of critical thinking skill will be evaluated, the nested single case design was used as the research design. Each situation considered in the nested multiple case design can be studied by dividing it into various subunits (Yıldırım and Şimşek, 2003). The context, situation and unit of analysis in the research design are shown in Figure 1.

CONTEXT
Critical Thinking Skills of Mathematics Teaching Undergraduate Students
SITUATION
Critical Thinking in Technology-Assisted Teaching of Quadratic Surfaces
Unit of Analysis
Critical Thinking Skills Analysis, interpretation, self-regulation, inference, explanation, and evaluation.

Figure 1. Nested Single-Case Pattern

Participants

In our study, students studying in the mathematics teaching undergraduate department constitute the population. For the sample, 2nd grade Primary School Mathematics Teaching students studying at a state university were selected by purposive sampling. Purposive sampling is a method that allows in-depth research by selecting information-rich situations depending on the purpose of the study (Büyüköztürk and all, 2018).

In the quantitative part of the study, 72 students from A and B branches who were studying in the 2nd grade undergraduate program of primary school mathematics teaching, who had received basic mathematics knowledge and who would learn the analytical geometry course in the same semester, were selected for the experimental and control groups.

In the qualitative part, which is the other part of the research, students were selected with the Criterion Sampling method (Büyüköztürk et al., 2018), which is one of the Purposive Sampling Methods that allows in-depth research by selecting information-rich situations depending on the purpose of the study. Critical Thinking Disposition (CTD) Scale (Semerci, 2016) was applied as a criterion to undergraduate mathematics teachers studying in both branches. According to the CTD Scale scores of each student, 10 students were selected for the experimental group and 10 students for the control group, considering them as low, medium or high.

Data Collection Tools

In our study, data was collected in two separate models, quantitative and qualitative, in order to examine the change in the critical thinking skills of primary school mathematics teaching undergraduate students. The Critical Thinking Disposition (CTD) Scale, revised by Semerci (2016), was used to collect quantitative data, and the Quadratic Surfaces Real Life Problems (QSRLP) prepared by the researcher was used to collect qualitative data. The Critical Thinking Disposition (CTD) scale consists of a total of 49 items, including meta-cognition, flexibility, systematicity, perseverance-patience and open-mindedness sub-dimensions. The CTD scale was applied as a pre-test and post-test. Critical thinking skills were used in the QSRLP activity. Facione's (1998) critical thinking skills of analysis, interpretation, self-regulation, inference, explanation, and evaluation were used in the QSRLP activity.

RESULTS

In the quantitative dimension of the research, the data obtained from the CTD scale were analyzed with the SPSS-22 package program. In determining the effect of independent variables (technology-supported quadratic surfaces teaching) on dependent variables (critical thinking), normality assumptions were tested in the experimental and control groups to determine the tests to be used in the analysis of the data. A normality test was performed on the data to determine whether the data was normally distributed. Since the data obtained from the CTD scale showed a normal distribution ($p>.05$) according to the results of the Kolmogorov Smirnov test, it was decided to use the parametric (normally distributed) unrelated samples t-test in comparisons between groups. In the qualitative dimension of the research, the data obtained by conducting interviews about the QSRLP effectiveness were analyzed by content analysis. The categories of the critical thinking skill theme were determined as analysis, interpretation, self-regulation, inference, explanation and evaluation, according to the critical thinking sub-skills determined by Facione (1998). It was evaluated according to these categories and verbatim quotes were made from the students' answers.

The pre-test scores of the students in the experimental and control groups, consisting of mathematics teaching undergraduate students, from the CTD scale were analyzed with the t test. The results obtained with the sub-dimensions of the CTD scale are given in Table 1. below.

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Table 1 . Independent Groups T-Test Results For The Critical Thinking Disposition (CTD) Pre-Test Scores of The Control (A) And Experimental (B) Groups

Sub-dimensions	Class	N	$\bar{X}$	ss	t	sd	p
Metacognition	A	33	3,89	0,39	1,39	68	0,16
	B	37	4,02	0,38			
Flexibility	A	33	3,83	0,44	1,77	68	0,81
	B	37	4,01	0,42			
Systematicity	A	33	3,71	0,39	1,80	68	0,08
	B	37	3,90	0,44			
Perseverance and Patience	A	33	3,60	0,57	1,33	68	0,19
	B	37	3,81	0,71			
Open-mindednes	A	33	3,66	0,62	1,14	68	0,25
	B	37	3,83	0,61			
Total	A	33	3,77	0,35	1,88	68	0,64
	B	37	3,94	0,41			

When the t-test analysis of the CTD scale, which we applied as a pre-test to the students in the control and experimental groups, was examined, looking at the CTD scale total score averages, the average of class A was found to be $\bar{X}_A$=3.77, and the average of class B was found to be $\bar{X}_B$=3.94. When CTD total total scores are examined between classes ($p=0.64$, $p>0.05$), it is seen that there is no significant difference.

When we examined according to the sub-dimensions of the CTD scale, students in the control and experimental groups had metacognition ($p=0.16$, $p>0.05$), flexibility ($p=0.81$, $p>0.05$), systematicity ($p=0.08$, It is seen that there is no significant difference in the scores of $p>0.05$), perseverance and patience ($p=0.19$, $p>0.05$) and open-mindedness ($p=0.25$, $p>0.05$). In general, it can be stated that the experimental and control groups in our study are similar, as there is no significant difference between the total scores of the CTD scale ($p=0.09$, $p>0.05$) that we applied to the experimental and control groups.

The students in the experimental group, in which technology-supported quadratic surfaces teaching was applied, and the control group, in which traditional teaching was applied,

Their posttest scores from the CTD scale were analyzed with a t test. The results obtained with the sub-dimensions of the CTD scale are shown in Table 2. below. It is given in .

Table 2 . Independent Groups T-Test Results For The Critical Thinking Disposition (CTD) Post-Test Scores Of The Control (A) And Experimental (B) Groups

Sub-dimensions	Class	N	$\bar{X}$	ss	t	sd	p
Metacognition	A	33	3,57	0,47	3,45	55	0,001
	B	24	3,95	0,39	3,56		
Flexibility	A	33	3,55	0,46	3,34	55	0,001
	B	24	3,99	0,49	3,30		
Systematicity	A	33	3,45	0,53	3,06	55	0,003
	B	24	4,11	1,07	2,77		
Perseverance and Patience	A	33	3,37	0,57	3,13	55	0,003
	B	24	3,88	0,62	3,09		
Open-mindednes	A	33	3,65	0,56	1,85	55	0,069
	B	24	3,95	0,66	1,89		
Total	A	33	3,51	0,44	3,62	55	0,001
		24	4,01	0,56	3,49		

Which we applied as a posttest to the students in the control and experimental groups , was examined, when looking at the CTD scale total score averages, the average of class A was found to be $\bar{X}_A$ = 3.51, and the average of class B was found to be $\bar{X}_B$ = 4.01. When the CTD total total scores $<$ between classes are examined (p=0,001 $p<<0,05$) a significant difference is seen. As a result of the comparison made to determine which groups the difference was in favor of, it was seen that technology-supported teaching was more effective in changing critical thinking tendencies than traditional teaching

In our study, critical thinking skills were evaluated in the QSRLPP activity, which includes real- life problems, according to Facione's (1998) categories of critical thinking skills: analysing, commenting, self-regulating, inferring, explaining and evaluating. Those who solved the questions completely, in a way that ensured their critical thinking skills, were evaluated as complete, those who solved the questions partially incompletely or incompletely, and those who could not solve them at all. Data were collected through the QSRLP activity obtained from both individual and bilateral interviews of students selected from the experimental and control groups . Interviews were held first with individual practice and then with bilateral practice.

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Questions at the QSRLP event The students' data were evaluated according to critical thinking skill categories. Some quotes from both individual and pair group interviews of students selected from the experimental and control groups are included.

The students codenamed Ela and Sena from the experimental group could not answer the question about the lighthouse that resembles a hyperboloid in their individual study. However, in the pair group interview, they were able to answer correctly the area of the shape whose base image is ellipse, which required evaluation skills under the guidance of the consultant teacher, using their prior knowledge under the guidance of the consultant teacher.

Ela and Sena : They said that the lighthouse given in the question resembles a hyperbola with a circle base , one of the geometric structures they have learned before.

Ela : Its base looks like a circle

Sena: But we can't know if it's a flat.

Ela : Yes, it could also be an ellipse, but at first glance he asks what it looks like, it looks like a circle.

Sena : This structure is three-dimensional. It looks like a single-winged hyperboloid

Then they read the question and answer it by evaluating it with the operation dimension, stating that since one of the given equations is negative, it is a one-winged hyperbola.

To find the image and area in the xoy plane

Ela : Let's draw the base on the xoy plane by giving z=0

Ela and Sena: "x=+6, x=-6 and y =+4 y=-4 are found, the base is ellipse" they find together

Ela : Its base turns out to be ellipse. We cannot find the area . We have not seen integrals in multivariable equations ($\frac{x^2}{36}+\frac{y^2}{16}=1$) $\frac{x^2}{36}+\frac{y^2}{16}=1$)

The teacher is called for help.

Teacher : What is its image in the xoy plane?

Ela : Ellipse.

Teacher : What is the feature of ellipse?

Ela: It has no radius

Teacher : What do you think about the points where the axes intersect?

Ela: 4 and 6

Teacher : What would happen if they were the same?

Ela Flat and $\pi r^2 \pi r^2$ it would have space.

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Teacher : What does the r here mean? **Ela :** Radius

Teacher : Say the area of the circle again **Ela** :$\pi r^2 \pi r^2$

Teacher : Try to make the area of the ellipse similar to this

Ela : Thinking and slowly.6 .4.$\pi\pi$ he stated.

Teacher: Yes, that's right.

Sena : Oh ,

Ela : Saying "I am enlightened" (They are surprised).

Ela and Sena: Then the area of the ellipse is π $r_1 r_2$. $\pi\pi$ . . It is found as 6.4 $\pi\pi$= 24 $\pi\pi$. We learned something new Highlight !

They were happy by saying .

Ela and Sena's individual and pair group solutions are shown in Table 3.

Table 3. Ela And Sena Question-2 Solutıon Critical Thinking Skills Table

Critical Thinking Skills	**Analysis**	**Interpretation**	**self-regulation**	**inference**	**Explanation**	**Evaluation**
Ela	X	X	X	X	X	
Sena	X	X	X	X	X	
Ela and Sena	X	X	X	X	X	X

In the fifth question, the concept of a rotating surface, which resembles a sphere as a quadratic surface, requires high-level thinking. In the fifth question, Ela explains the concept of a rotational surface, which requires high-level thinking, both algebraically and geometrically in Figure 2. also explained .

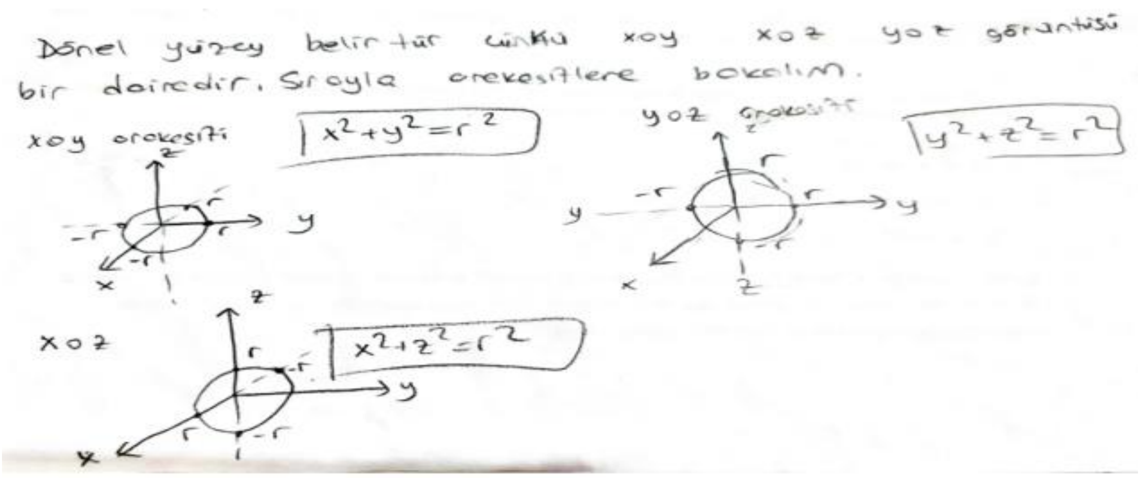

Figure 2. Ela's Solution to the Surface of Revolution Question

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Different quadratic surfaces is examined, the student named Osman in the experimental group explained that the first part of the given shape resembles an elliptical cone (analysis, interpretation surface recognition), and stated that the unique pointed tip of the cone will be created by cutting the lines drawn on the ellipse-based side faces at the origin (separation into its components) (Figure 4.).

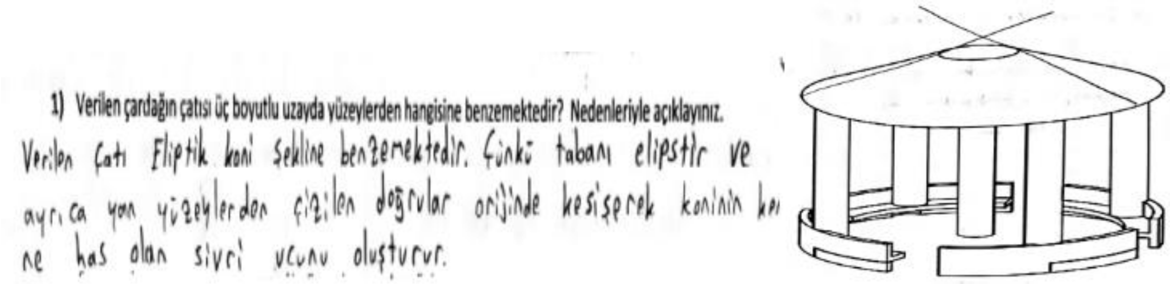
1) Verilen çardağın çatısı üç boyutlu uzayda yüzeylerden hangisine benzemektedir? Nedenleriyle açıklayınız.
Verilen çatı Eliptik koni şekline benzemektedir. Çünkü tabanı elipstir ve ayrıca yan yüzeylerden çizilen doğrular orijinde kesişerek koninin kendine has olan sivri ucunu oluşturur.

Figure 4. Osman's Solution For Question-2

In the question, he was able to find the appropriate equation by associating it with appropriate quadratic surfaces and explain the reason (Self-organization, inference). In the second part of the pergola question, he realized that if he wanted to cover the pergola with nylon, the shape that would be formed would be an elliptical cylinder. By visualizing it correctly in his mind, he was able to find the appropriate equation and draw the correct shape (infer). He showed that the appropriate quadratic surface equation would have an ellipse as its base (a$> b) > b)$ ellipse would be different. In the question, he explained the surface specified by each equation in two and three dimensions and explained it correctly in Table 4. (Explanation).

Table 4. Osman' Solutıon of Question-2 Critical Thinking Skills Table

Critical Thinking Skills	Analysis	Interpretation	self regulation	inference	Explanation
Osman	X	X	X	X	X

The student named Begüm in the experimental group stated that the lighthouse given in the third question in the (QSRLP) activity resembles a hyperbola (Analysis, Interpretation), and its baseresembles an ellipse, among the geometric structures he has previously learned. To create this shape,he associated it with the appropriate hyperbola curve (Self-organization), and correctly drew that the hyperbola curve could be created by rotating it around the z-axis (separating it into its components, rotating, moving), and found that he could associate it with appropriate quadratic surfaces (single winged hyperboloid) (Inference). It is stated below (Figure 4.).

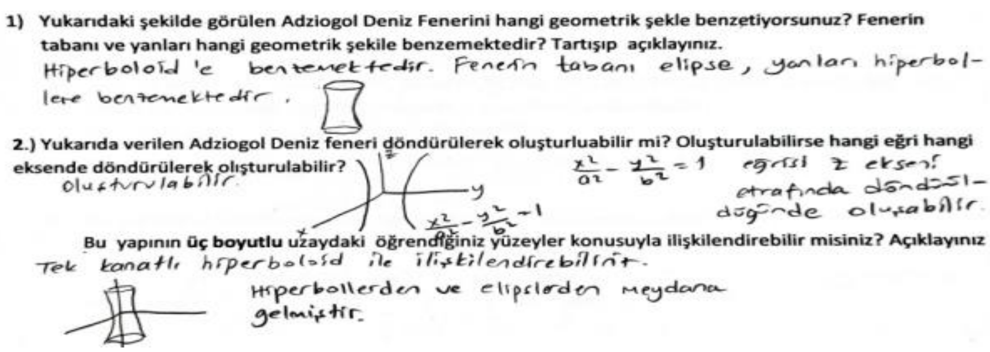
1) Yukarıdaki şekilde görülen Adziogol Deniz Fenerini hangi geometrik şekle benzetiyorsunuz? Fenerin tabanı ve yanları hangi geometrik şekile benzemektedir? Tartışıp açıklayınız.
Hiperboloid'e benzemektedir. Fenerin tabanı elipse, yanları hiperbollere benzemektedir.
2.) Yukarıda verilen Adziogol Deniz feneri döndürülerek oluşturluabilir mi? Oluşturulabilirse hangi eğri hangi eksende döndürülerek oluşturulabilir?
Oluşturulabilir.
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ eğrisi z ekseni etrafında döndürülerek oluşabilir.
Bu yapının üç boyutlu uzaydaki öğrendiğiniz yüzeyler konusuyla ilişkilendirebilir misiniz? Açıklayınız
Tek kanatlı hiperboloid ile ilişkilendirebilirim.
Hiperbollerden ve elipslerden meydana gelmiştir.

Figure 4. Begüm's Solution For Questıon-3

His groupmate Osman also gave correct answers to this question. However, both of them could not find the answer to the question of calculating the floor area together. However, with the guidance of the consultant teacher, they were able to find the area of the shape with an ellipse base. Their critical thinking skills table is given in Table 5.

Table 5. Begüm and Osman Group's 3rd Question Critical Thinking Skills Table

Critical Thinking Skills	Analysis	Interpretation	self-regulation	inference	Explanation	Evaluation
Begum	X	X	X	X	X	
Osman	X	X	X	X	X	
Begüm&Osman	X	X	X	X	X	X

Students codenamed Begüm and Osman from the experimental group are asked a question that requires high-level thinking. "Which surfaces can be specified in R^3 dimension of a shape whose image is an ellipse in R^2 dimension?" They answered the question correctly in Figure 3. by explaining all possible situations.

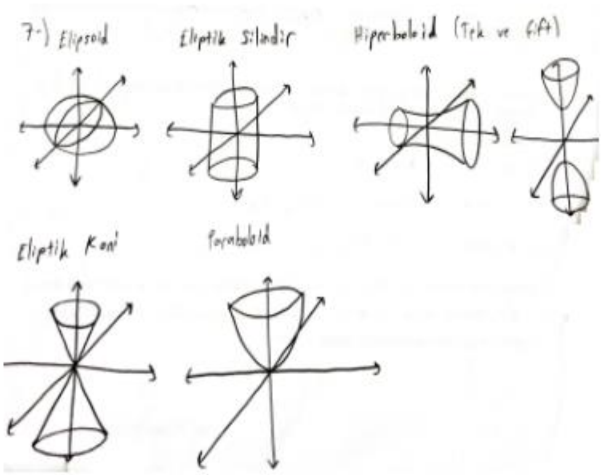

Figure 3. Begüm and Osman's solution for Question

These two students, who have a certain tendency to think critically, complemented each other while solving the question together and achieved a high level of critical thinking skills (Table 3.).

Table 3. Begum and Osman question 2 critical thinking skills table

Critical Thinking Skills	Analysis	Interpretation	self-regulation	inference	Explanation	Evaluation
Begum-Osman	X	X	X	X	X	X

The students in the experimental group, Ramiz and Zehra, reached the evaluation stage in their similar individual studies on the lantern question, they could not find the area of the ellipse, which is the base image of the lantern. The study was carried out in pairs under the guidance of the consultant and the desired result was achieved and both algebraic and geometric explanations were made in Figure 5.

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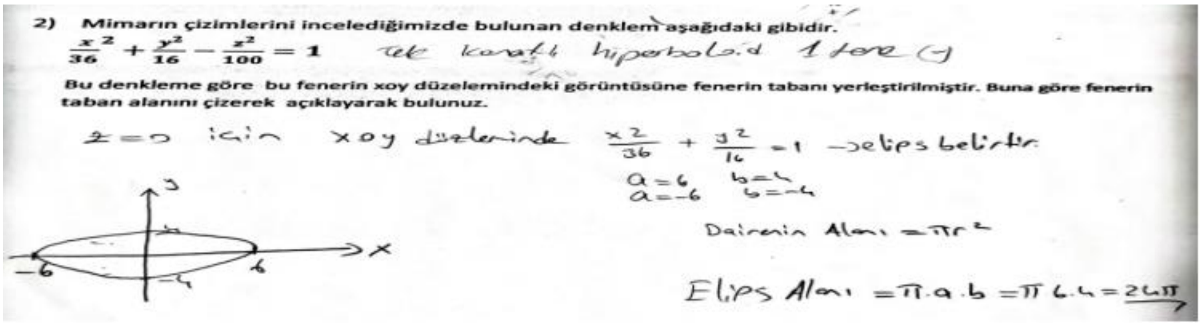

Figure 5. Zehra and Ramiz 3.question solution of evaluation step.

Of the duo Ali and Esra, who had similar critical thinking levels in the control group, was examined. While Esra was inadequate in the individual study, Ali, who answered the question correctly in the pair study, made the necessary explanations to his friend and recognized the shape by stating that the shape resembles a single-winged hyperbola, touching on the concept of width and height, and stating that the side surfaces are hyperbola (analysis). A mistake was made in interpreting that it resembles a single-winged hyperbola instead of calling it a single-winged hyperboloid. When they examine the equation given in the question, they find the points where the image of the single-winged hyperboloid on the xoy plane intersects the axes of the ellipse (Explanation) and the shape of the ellipse is drawn correctly (Drawing the shape). They were able to find the area after calling the teacher and getting help to find the base area (Table 6) .

Table 6. Ali and Esra Critical Thinking Skills Table

Critical Thinking Skills	Analysis	Interpretation	self-regulation	inference	Explanation	Evaluation
Ali	X		X	X	X	
Esra	X					
Ali and Esra	X		X	X	X	X

Solutions that could not be solved through individual studies were solved with the help of the teacher. Additionally, it was observed that mathematical language was used incorrectly in the duo's work.

DISCUSSION and CONCLUSIONS

First of all, according to the results of the CTD scale that we applied to the experimental and control groups, the critical thinking tendency average of the students in the control group was found to be $\bar{X}_A$ =3.77, and the critical thinking tendency average of the students in the experimental group was found to be $\bar{X}_B$ =3.94. According to these averages, it can be said that the critical thinking tendencies of students in both classes are high.

There are also supporting studies in parallel with the research results in which the critical thinking tendencies of mathematics teaching undergraduate students are high. Kutluca, Yılmaz and İbiş (2018) The critical thinking attitudes of teacher candidates studying in different departments of the faculty of education affiliated to a foundation university in Istanbul are above the average level. Karalı (2012) stated that teacher candidates' critical thinking disposition levels were at a "good level"; Tunçer and Sapancı (2021) found that secondary school mathematics teachers' critical thinking tendencies were at a good level; Korkmaz's (2009) studies in which teachers in the numerical subject group working in primary, secondary and higher education institutions have a high critical thinking tendency coincide with our study in which the critical thinking tendencies of primary school mathematics teaching undergraduate students, who are prospective mathematics teachers of the future, are found to be high.

After technology-supported teaching using Geogebra software, the CTD scale was applied to both the experimental and control groups. When the t-test analysis of the CTD scale, which we applied as a post-test to the students in the control and experimental groups, was examined, when looking at the CTD scale total score averages, it was found that the average of Class A in the control group was $\bar{X}_A$ = 3.51, while the average of Class B in the experimental group was $\bar{X}_B$= 4.01. When the CTD total total scores between classes are examined ($p = 0.001$, $p \ll 0.05$), a significant difference is seen. As a result of the comparison made to determine which groups the difference was in favor of, it was seen that technology-supported teaching was more effective in changing critical thinking tendency positively than traditional teaching . The results obtained indicate that Bayram's (2015) WebQuest -supported applications using technology positively affect the critical thinking skills of prospective English teachers . Gürsan , Topan Broutin and İpek's (2021) study with technology-supported teaching practices using GeoGebra software to increase critical thinking skills with prospective teachers and Alp's (2019) positively improved students' conceptual understanding and critical thinking skills with learning based on Web Supported Collaborative Learning with the Scratch Program. It is similar to the study of Schreglmann and Karakuş (2017), which found that students using educational software supported by an educational interface with critical thinking teaching features increased their critical thinking skills.

In the real-life example given in the QSRLP activity , it is seen that although "circular floor" is stated in the question stem, those who cannot read meaningfully describe quadratic surfaces as elliptical cylinders. It was observed that those in the experimental group who read the question meaningfully were successful in recognizing and naming the shape, analyzing its components and interpreting the shape. Despite the shape definitions, it was observed that the students in the control group were inadequate in explaining its difference from the other surfaces given in the question and in drawing its shape in two dimensions, while the students in the experimental group had fewer problems in these steps.

Different quadratic in the second question, which is the combination of surfaces , the information that the base surfaces will be common when the roof with an ellipse base and the surface formed when the surface is closed with nylon are combined, could not be combined and visualized. In particular, although the root of the question stated a roof with an ellipse base, it was observed that the wrong answer was given by choosing the surface with a circular base, as no meaningful reading could be made. In both groups, students with medium and high critical thinking tendencies preferred to make algebraic explanations instead of geometric explanations. However, it was observed that the students in the experimental group were more successful in drawing the figure and indicating the point where it intersects the axes, in addition to solving the algebraic operation.

The single-winged hyperboloid surface was given by comparing it to a cylindrical surface. In this question, where we want to recognize the hyperbola curve given in the question and visualize its rotation with the y axis, we are expected to first recognize the shape, find the appropriate equation, and then calculate the area of the base image of this shape. A suitable hyperbola curve was found in both groups. Most of the students in the experimental group explained that the base image would be an ellipse and the points where it intersects the axes, both with an algebraic solution and by drawing the points where the ellipse intersects the axes. Students in the control group generally preferred to explain only with algebraic solutions. They were inadequate in drawing the desired shape. Although the base image was found to be an ellipse, they stated that "we do not know how to find the area of the ellipse." In the group work, with the support of the teacher, the answer to the question was found by utilizing the department's field knowledge. In short, the correct solution to the problem could be made by associating preliminary information with the question. It was also observed that in this question, the wrong concept was used as "single-winged hyperboloid " instead of the expression "single-winged hyperboloid", which is a three-dimensional surface.

Students in both groups were able to correctly name the sphere shape, which has been recognized for years, and associate it with the appropriate quadratic surface. It is seen that they can analyze the similarities and differences between circles and ellipses. We can say that the students in both groups were successful in solving this problem, which they could easily draw with Geogebra . In other words, the solution of the questions does not change in both groups until the analysis, interpretation and self-regulation steps. Although the students in the control group knew the situation in the question in which we measured the concept of rotational surface in the evaluation step, they had difficulty in making inferences about this situation based on the data. It was observed that the students in the experimental group evaluated the question by inferring the explanations based on their prior knowledge. As a result, it is seen that the differentiation between the groups has changed from critical thinking skills to inference, explanation and evaluation steps that require high-level thinking, and this change is positive in favor of the experimental group.

Finally , in the question in which we asked the students to make a judgment that they could create the appropriate surface in R^3 by using their prior knowledge and visualizing the image being an ellipse , they were able to draw the most ellipses and elliptic cylinders in both groups. It was observed that students in the experimental group with a high tendency to think critically were more successful in finding all possible situations and drawing their shapes. There are studies that reveal the relationship between critical thinking skills and academic success. In general, it is similar to studies stating that there is a positive relationship between students' achievements and critical thinking skills (Kürüm, 2002; Varaki , 2006; Kalkan, 2008).

In general, even if students with low and medium levels of critical thinking tendencies recognized the shape and solved the question, they were insufficient to explain their reasons and only wrote the answer. Especially among the students in the experimental group taught with geogebra , students with a high tendency to think critically explained the judgments requested in the question with justifications and in a consistent manner, and made inferences both algebraically and with figure drawings, especially with convincing arguments . As a result of their study, Sumartini and Maryatini (2021) stated that the conceptual and procedural learning of quadratic functions of the students in the experimental group, where the subject was taught with Geogebra , was more effective. When the answers of the students in the experimental and control groups are evaluated, there is no difference in the analysis, interpretation and self-regulation steps of critical thinking skills in solving the questions. It is seen that critical thinking skills have changed in the inference, explanation and evaluation steps that require high-level thinking, and this change is positive in favor of the experimental group. It can be said that the students in the experimental group, who were taught with Geogebra , were more successful in solving questions with critical thinking skills, especially in the inference explanation and evaluation step, which requires higher-level thinking.

That the abstract algebraic expressions on quadratic surfaces studied in the analytical geometry course were concretized with the help of Geogebra , thus improving students' visualization and critical thinking skills. As a result, we can say that the use of the dynamic software Geogebra in the educational environment enables the positive development of students' critical thinking skills and increased success. Studies have shown that the use of Geogebra and similar dynamic software will help students recognize geometric objects, discover their properties, develop their visual thinking skills, support generalization and reasoning skills and increase success (Christou , Pittalis , Mousoulides , Pitta , Jones, Sendova, & Boytchev , 2007; Özen, 2009). ; Kepçeoğlu, 2010; Ceylan, .2015 ; Çörekçioğlu, 2019, Hawes ., Gilligan -Lee, and Mix , 2022).

Suggestions

In today's world where technology is rapidly advancing, critical thinking emerges as a necessary life skill to survive in the 21st century (Bybee and Fuchs , 2006). Applications of Geogebra software , which is used in technology-supported education that improves critical thinking , can be found in high school textbooks, slope networks (EBA) and other social media networks. Studies indicate that teacher candidates in our country are not adequately prepared to teach critical thinking to their students and that teacher training programs need to be organized accordingly (Karadeniz, 2006; Alkın, 2012; Yeşilpınar and Doğanay, 2014). As stated by Clarke (2009), prospective teachers need to be trained to use technological skills in order to see the power of technology in increasing mathematical knowledge.

Analytical geometry course is one of the important courses in which both algebraic knowledge and geometric knowledge are used. Quadratic surfaces correspond to the algebraic expansions in analytic geometry of some shapes we see around us in daily life. While preparing for the study , it was observed that there is generally theoretical knowledge and applications in the national literature on the subject of quadratic surfaces , which is taught in the analytical geometry course in the Mathematics teaching undergraduate program . When researched on a national scale, there were technology-supported applications on the subject of integrals, but no sufficient resources were found on technology-supported applications on the subject of quadratic surfaces. For this reason, foreign sources were used to draw and explain surfaces such as ellipsoid , paraboloid and hyperboloid , which are taught about quadratic surfaces, using dynamic software. Software that can be applied in mathematics and analytical geometry courses in undergraduate education should be integrated into the courses and application examples should be increased. Technology is constantly changing rapidly. As new technological developments occur, technological tools used in the educational environment need to be updated and adapted to education.

It is important that mathematics teaching undergraduate students, who will educate future generations, have sufficient equipment in the applications of technology in education. For this reason, the number of courses that will improve the usability of technology in education should be increased in undergraduate programs. Environments should be provided where students can practice one-on-one in these courses.

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