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MATHEMATICAL THINKING PROCESS OF 6TH GRADE STUDENTS IN ACCORDANCE WITH KISS THEORY[1]

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Abstract

In this study, the mathematical thinking skills of sixth grade students were examined in terms of KISS theory. The term KISS stands for conceptual operational symbolic process. KISS theory is based on the procept theory developed by Gray & Tall (2004). KISS theory consists of three stages: the procedure, the operational process and the conceptual operational symbolic process. The research was designed as a case study method. Students' mathematical thinking skills were examined during the transition from arithmetic to algebra. The research was carried out with five sixth grade students. The students were selected on the basis of their average performance in the mathematics course. It was found that one student had a low level of achievement in mathematics, two students had a medium level of achievement and two students had a high level of achievement. The students had previous education on algebraic expressions. Semi-structured interview technique was used in the research. Five open-ended questions aimed at revealing their knowledge of arithmetic and algebra were asked to them. The descriptive analysis method was chosen as the data analysis method. It was seen that the sixth grade students were able to think correctly in the procedure and the operational process and the expressing the concept symbolically. Students were generally able to write correct symbolic expressions for verbal expressions. However, while explaining symbolic expressions, they generally preferred to give numerical values instead of unknowns. Therefore, they first realized concrete thinking. As a result, the students first carried out a process of thinking about the procedure and the operational process. Then, they made the transition to the stage of symbolic thinking by considering the concept and the operation together.

Keywords: KISS theory, mathematical thinking, sixth graders.

INTRODUCTION

In the process of solving the problems they encounter in daily life, individuals think about how to solve this problem. Likewise, a thinking process takes place in solving mathematical problems. The thinking that takes place here is the mathematical thinking process in which mathematical skills are used. Some of these mathematical skills are reasoning, problem solving, prediction, using and transforming multiple representations, and abstraction**.** Mathematical thinking includes higher order thinking skills (Tall, 2002). When the literature is examined, mathematical thinking is defined by some researchers (Alkan & Bukova Güzel, 2005; Arslan & Yıldız, 2010; Bulut, 2009; Burton, 1984; Dreyfus, 1990; Henderson et al., 2002; Keskin, Akbaba-Dağ & Altun, 2013; Liu, 2003; Liu & Niess, 2006; Ma'Moon, 2005; Mason, Burton & Stacey, 2010; Polya, 1945; Stacey, 2006; Tall, 2002; Yeşildere, 2006; Yeşildere & Türnüklü, 2007) on the basis of skills such as prediction, generalization, induction, deduction, modeling, reasoning, customization, using symbols, and abstract thinking; it is defined by some researchers (Schoenfeld, 1992; Tall, 2006) as the process of formation of concepts in the mind. The individual uses mathematical thinking processes to make sense of a concept (Schoenfeld, 1992). The individual tries to

[1] The article is produced from the first writer's doctoral thesis in consultation of the second author.

understand the new concept within existing knowledge structures. The skills he/she uses in this understanding process are mathematical thinking skills. The mathematical thinking process is individual. The skills used in the process vary from person to person. Therefore, it should not be expected that certain rules will be applied in mathematical thinking (Henderson et al., 2002). It should not be thought of as simply explaining the answer to a mathematical problem. Being able to develop and demonstrate a thinking process regarding the problem solving process requires mathematical thinking. In other words, mathematical thinking does not mean dealing with mathematics. It is the ability to improve the thinking process even in problems encountered in daily life.

Students have difficulty in transitioning from concrete thinking to abstract thinking (Arslan & Yıldız, 2010; Keskin, Akbaba Dağ & Altun, 2013; Yeşildere & Türnüklü, 2007). Tall (2002) argues that as abstract thinking skills increase, mathematical thinking skills will also increase. The individual's existing knowledge structures are important in the transition to abstract thinking. The individual uses existing knowledge structures in the process of learning the concept. In this sense, concrete thinking is important in the transition to abstract thinking. Therefore, it is important to reveal existing knowledge structures when examining an individual's process of expressing a concept. In this research, students' mathematical thinking skills were evaluated through the transition process from arithmetic to algebra.

KISS theory, which forms the theoretical framework of the research, is based on the process-object interaction, which Gray and Tall (1994) considered as a procept. Gray and Tall (1994) defined the symbolic representation of the concept and operational process as procept. Procept is a combination of the words process and concept. According to procept theory, concept formation occurs as a result of an operational process. If the individual can represent the concept symbolically as a result of the operational process, it means that he/she has reached the procept level in terms of mathematical thinking processes. Procept theory emerged with the concept of number (Gray & Tall, 1994). When the concept of number is examined, it includes both the counting process and the symbolic expression resulting from the counting process. For example, if the number 3 is considered, it represents both the counting process as 1,2,3 and the number 3 that emerges as a result of the operational process. The counting process is important in the formation of the concept of number (Fuson & Hall, 1983; Gelman & Meck, 1986; Wagner & Walters, 1982). A similar situation is valid for the algebraic expression "3a+b". It shows "b more than 3 times the number a". Therefore, it is the symbolic expression of both the operational process and the concept that emerges as a result of the process. The basis of KISS theory is based on procept theory. KISS consists of the initials of the words conceptual operational symbolic process in Turkish. KISS theory is based on these four components. In mathematical thinking processes, arithmetic and algebraic skills and expressing with symbols are important. In this theory, mathematical thinking stages have three stages. These are transaction, operational process and conceptual-operational symbolic process stages, as in procept theory. Gray and Tall (1994) determined the skills of these stages as follows:

Procedure: This is the phase in which the student learns the operation. Gains knowledge about how to apply the procedure. The student follows the step-by-step instructions. The action taken may be meaningless for him at this stage.

Operational process: It is the phase in which the process is repeated. The knowledge acquired as a result of constant repetition begins to emerge. Repeated operations begin to make sense for the student.

Conceptual-operational symbolic process: This is the phase in which the student, who learns the application of the process as a result of repetitions, can make a judgment about the result without applying the process steps. The student explains the concept with its features. He/she expresses the concept symbolically.

There are studies in the literature that examine the mathematical thinking processes of individuals in terms of different abstraction theories such as APOS Theory (Açan, 2015; Açıl, 2015; Deniz, 2014;

Ercire, 2014; Günaydın, 2018; Gürbüz, 2018; Hannah, Stewart & Thomas, 2016; Hazar, 2021; Martínez-Planella & Triguerosb, 2019; Mudrikah, 2016, Şefik, 2017), RBC Theory (Altaylı Özgül, 2018; Bayraktar, Aydoğdu & Tutak, 2022; Dinç & Yenilmez, 2022; Eldekçi, 2019; Kaplan & Açıl, 2015; Tabach & Hershkowitz, 2002; Tsamir & Dreyfus, 2002; Türnüklü & Özcan; 2014; Ulaş & Yenilmez, 2017; Yeşildere, 2006), and Solo Taxonomy (Bağdat & Saban, 2014; Chan et al., 2002; Çelik, 2007; Groth & Bergner, 2006; Köse, 2018; Lian, Yew & Idris, 2006; Lucas & Mladenovic, 2008; Rider, 2004). Some studies have also examined mathematical thinking in terms of procept theory (Chin & Tall, 2002; Kidron, 2008; Watson, Spyrou & Tall, 2003). In addition, as a result of the literature review, it was seen that studies examining mathematical thinking processes generally focused on the awareness of teachers and teacher candidates (Alkan & Tataroğlu Taşdan, 2011; Baş, 2013; Casey & Amidon, 2020; Flake, 2014; Goggins, 2007; Heng & Sudarshan, 2013; Jacobs, Lamb & Philipp, 2010; Kükey, 2018; Lane, 2005; Liu, 2014; Özdemir Baki & Işık, 2018; Tataroğlu Taşdan, Çelik & Erduran, 2013; Tohir et al., 2020, Uygun, 2020). On the other hand, it has been determined that some studies have examined the mathematical thinking skills of secondary school students (Bahadır, 2020; Baltacı, 2016; Benli & Gürtaş, 2021; Bozkurt & Topal, 2019; Cai, 2000; Cai, 2003; Gürbüz, 2021; Karakoca, 2011; Karslıgil Ergin, 2015; Kılıç, Tunç Pekkan & Karatoprak, 2013; Kükey, 2018; Sezer, 2019; Sezgin Memnun, 2011; Tüzün & Cihangir, 2020; Yeşildere, 2006; Yeşildere & Türnüklü, 2007; Yıldırım & Yavuzsoy Köse, 2018; Zhao, Alexander & Sun, 2020). This research focused on the mathematical thinking processes of sixth grade students. Cai (2000), one of the researchers who examined the mathematical thinking skills of sixth grade students, discussed these skills of students in America and China in the context of open-ended and closed-ended questions. While students in China were more successful in closed-ended questions, students in America were more successful in open-ended questions. Karakoca (2011) determined that sixth grade students had difficulty in skills such as flexible thinking and reasoning during mathematical thinking. Students were more successful in routine questions than those who did not. Sezgin Memnun (2011) examined the mathematical thinking of sixth grade students in terms of RBC+C theory. It has been determined that students' mathematical thinking is supported by realistic mathematics education and educational environments organized with a constructivist approach. Kılıç, Tunç Pekkan and Karatoprak (2013), in their research examining the effect of material-supported teaching on sixth grade students' mathematical thinking, found that students' misconceptions created difficulties in understanding the activities. Bozkurt and Topal (2019) discussed the mathematical thinking processes of sixth grade students in the context of problem solving. As a result of the research, it was seen that students had problems in solving problems due to errors in using visual drawings, numerical symbols and verbal representations. The small number of studies conducted with sixth grade students in the literature (Bozkurt & Topal, 2019; Cai, 2000; Karakoca, 2011, Kılıç, Tunç Pekkan & Karatoprak, 2013; Sezgin Memnun, 2011) was effective in determining the study group as these students.

The specific objectives of the secondary school mathematics curriculum (MEB, 2018) include students' ability to understand and use mathematical concepts. Students must demonstrate these skills within mathematical thinking processes. In the light of all this information, the basis of this research is to examine the mathematical thinking skills of sixth grade secondary school students in the transition process from arithmetic to algebra. In addition, it is thought that examining students' mathematical thinking skills in the context of the process in terms of KISS theory constitutes the originality of the research.

METHOD

Since the mathematical thinking processes of sixth grade students were examined on the basis of KISS theory, a case study was chosen as the qualitative research design. Case study is a preferred method in cases where "why" and "how" questions become important (Yin, 2003). Explanatory/descriptive case study (Yin, 2003) was used in the research because it involves sharing descriptive information about a situation. In this method, cause and effect relationship is important. In this research, the mathematical

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thinking processes of sixth grade students, their prior knowledge and the way they express concepts based on their prior knowledge were examined.

Participants

Purposeful sampling method was used to determine the study group of the research. Criterion sampling design was chosen from this method. In the method, the criteria are created by the researcher (Patton, 2014). In the study, it was determined that sixth grade students had received instruction on the achievements of algebraic expressions and had prior knowledge of the concept of algebra. Five sixth grade students studying in public secondary schools in İzmit district of Kocaeli province were determined as the sample. As a result of obtaining the necessary permissions, the study was able to be conducted with one student with a low level of success, two students with an intermediate level and two students with a high level of success.

The previous year's report card grades were taken into consideration in determining the students' mathematics course success averages. The student with a low level of success was named S1 (grade average 47), the students with a medium level were named S2 (grade average 73) and S3 (grade average 81), the students with a high level were named S4 (grade average 98) and S5 (grade average 100). While coding student names, they were coded from low to high, from S1 to S5, according to their success levels.

Collection of data

In this study, sixth grade students' mathematical thinking skills in terms of KISS theory were discussed in the context of the transition process from arithmetic to algebra. Semi-structured interviews were conducted with the students. In this technique, questions are prepared in advance. These questions are then asked to the participants. It is a method that provides flexibility to the researcher (Türnüklü, 2000). Data were collected with five open-ended questions prepared by the researchers. The first of these questions is "Can you tell me about 5?". It is aimed to reveal students' knowledge of the concept of number. The second question is "Can you explain 4+3?". In this question, they were asked to explain the operation and the meaning it expresses. The third question is "Can you explain a+3b?" was determined as. In this question, they were expected to express their knowledge of algebraic expressions. The fourth question is "Can you compare the expression 'First multiply a number by 4, then add 8' with the expression 'Add 2 to the same number and then multiply by 4'? Can you explain it algebraically?". In this question, students were expected to create symbolic expressions appropriate to verbal expressions. Finally, "'If x is even, then 2x+5 is odd.' Can you explain this statement?" was asked. This question aims to reveal the skills of interpreting algebraic expressions.

Analysis of data

Descriptive analysis method was chosen as the data analysis method. In this method, data are examined within the framework of predetermined themes (Yıldırım & Şimşek, 2011). The triangulation method (Creswell, 2013) was used to ensure validity and reliability. Documents containing interview records and student responses were examined in the light of the themes determined by the researchers. Researchers classified the skills of three mathematical thinking processes (procedure, operational process, conceptual-operational symbolic process) of the KISS theory in line with student answers. By making separate evaluations regarding which mathematical thinking process the student answers belonged to, the percentage of agreement between the two researchers was determined. In qualitative data analysis, evaluations of more than one researcher should be compared (Miles & Huberman 1994). The calculated agreement percentage was found to be 83%. Since this percentage value is over 70% (Miles & Huberman, 1994), the data evaluation process is considered reliable.

RESULTS

In the research, the students were first asked "Can you tell me about 5?". Since the concept of "5" includes both the counting process and the concept, the question aims to determine students' knowledge of 5 in terms of KISS. The data are as in Table 1.

Table 1. Mathematical Thinking Processes of 6th Grade Students Regarding the Concept of "5"

KISS levels	Skills	S1	S2	S3	S4	S5
Procedure	Obtain by counting		χ		χ	
	4+1, 3+2 express as			χ		χ
Operational process	The number after 4, the number before 6, etc.		χ			
Conceptual operational symbolic process	Stating that 5 will be obtained at the end of the operational process		χ	χ	χ	χ

S1 said 5 as the integer number. He/she stated that when creating a square, its sides could be 5 cm. He/she showed it symbolically by writing 5. On the other hand, he/she could not explain the procedure and operational process regarding 5. S2 said that the fifth of the digits, consecutive to four, is a number that we can obtain when we count from one. S3 stated that 5 is a number formed as a result of addition or subtraction. S4 stated 5 as an odd number and a natural number. He/she said that 5 could be obtained by counting. S5 stated that when we add 3 and 2, we can get 5. He/she also stated 5 as an odd number and a prime number. While explaining 5, this student said, "If we add 2 and 3, we get 5. It is an odd number. It is a prime number. If we add 1 and 4, we get 5."

In order to reveal the students' ability to think together about the operation and the concept resulting from the operational process, "Can you explain 4+3?" was posed. The data are as in Table 2.

Table 2. Mathematical Thinking Processes of 6th Grade Students Regarding the Concept of "4+3=7"

KISS levels	Skills	S1	S2	S3	S4	S5
Procedure	Count 5,6,7 after 4					
	Adding 3 to 4				χ	
Operational process	The sum of the numbers 4 and 3	χ	χ	χ		χ
Conceptual operational symbolic process	Stating that 7 is the result of addition	χ	χ	χ	χ	χ

Regarding "4+3", S1 stated that they were asked about their sum and said that the result would be 7. He/she explained that it could be the sum of 4 steps and 3 steps. S2 stated that the answer would be 7 as a result of the addition process. S3 stated that the number 7 is formed as a result of the sum of the numbers 4 and 3. S4 said that when 3 is added to 4, 7 will be obtained. He/she was able to state that it was an addition operation and the result was 7. While explaining 4+3, this student said, "The sum of the number 4 and 3 is 7. For example, Ali has 4 liras. His father also gives 3 liras." He gave an example as follows. S5 stated that the number 7 is obtained as a result of the sum of the numbers 4 and 3. As can be seen, all students were able to think symbolically about the concept formed as a result of the operational process.

During the transition from arithmetic to algebra, in order to examine students' algebraic knowledge structures in terms of KISS, we asked them "Can you explain a+3b?". The data are as in Table 3.

Table 3. Mathematical Thinking Processes of 6th Grade Students Regarding the Concept of "a+3b"

KISS levels	Skills	S1	S2	S3	S4	S5
Procedure	Giving concrete examples from daily life		χ	χ		χ
Operational process	The sum of a and 3 b's		χ			χ
Conceptual operational symbolic process	The sum of one unknown number and three times another unknown number		χ	χ		χ
	Algebraic explanation		χ			

S1 stated that "a+3b" is used in integers. He was able to say that letters were used instead of numbers. He/she stated his/her verbal expression as "3 more than a number". However, he/she said that the letter a would represent the number 3 and that a+3b would represent 6 from 3+3. S2 said that this expression will be obtained when we multiply 3 and b and add a. He expressed a and b as different unknowns. He/she gave an example from daily life as "the sum of one television brand and three other television brands.". S3 said that 3 is a constant and a and b are unknowns. He/she stated that a and 3b cannot be added together because they are different variables. This student interpreted the statement as “There are unknowns. We need to collect these unknowns. a and b are variables. a and b cannot be the same. b multiplied by 3. “The sum of one book and three different books.” explained as follows. S4 said that a and b are different variables. However, he/she could not explain the expression a+3b. S5 stated that a and b represent different numbers and 3b is 3 times b. While giving an example, he/she used the sentence "There is one quadrilateral and three triangles b". He/she was able to express the algebraic expression correctly verbally. However, he/she said that this statement has its consequences.

In order to examine the thinking processes that occur in the process of transforming verbal expressions into symbolic expressions, in terms of KISS, "Can you compare the expressions 'First multiply a number by 4, then add 8' with the expressions 'Add 2 to the same number, then multiply by 4'?" was asked. The data are as in Table 4.

Table4. Mathematical Thinking Processes of 6th Grade Students on Transforming Verbal Expressions into Symbolic Expressions and Comparing Skills

KISS levels	Skills	S1	S2	S3	S4	S5
Procedure	Experiment with numerical expressions	χ	χ	χ	χ	χ
Operational process	Generalizing operations		χ	χ	χ	χ
Conceptual operational symbolic process	Algebraic expression		χ		χ	χ
	Algebraic explanation					

S1 stated that these expressions are the same. He/she showed that the reason for this was that the words add and multiply were used in both expressions. He/she showed that when he/she thought of a number as the number 4, the numerical results of two expressions were equal. When asked to write algebraic expressions, he/she expressed them in different notations. He/she interpreted this situation as "the results are the same, but the algebraic expressions are different." S2 said that the expressions were different because one added 2 and the other added 8. When he/she tried using numbers instead, he/she showed that the results were the same. He/she was able to write correct algebraic expressions suitable for the expressions. However, he/she said that the equality of two algebraic expressions can be shown when the value of x is known. S3 stated that there should be parentheses in the second expression and that the two expressions are different. However, when he/she wrote a number instead of x, he/she showed that the results were equal as a result of the operational process. He/she explained this equality by the fact that their x's, that is, their letters, are the same. When S4 thought of a number as 3, he/she was able to show the equality of two expressions as a result of numerical operations. When he/she wanted to write algebraic expressions about expressions, he/she was able to write correct algebraic expressions; However, he/she could not explain their equality algebraically. S5 stated that when x is given a numerical value, the two expressions will be equal. He/she was able to show correct algebraic expressions for two expressions. He/she could not explain his/her equations algebraically..

Finally, "'If x is even, then 2x+5 is odd.' Can you explain this statement?" was asked. In this question, students were expected to explain algebraic expressions symbolically. Data regarding student answers are as in Table 5.

Table 5. Mathematical Thinking Processes of 6th Grade Students Regarding Their Skills in Explaining Symbolic Expressions

KISS levels	Skills	S1	S2	S3	S4	S5
Procedure	Experiment with numerical expressions	χ	χ	χ	χ	χ
Operational process	Generalizing operations			χ	χ	χ
Conceptual operational symbolic process	Algebraic explanation					

S1 said that if x is an even number, the result cannot be an odd number. He/she stated that if x = 44, the sum of 44 and 5 would be 49. Lastly, he/she wanted to multiply 49 by 2 and said that the result would be an even number, thus defending that the statement was wrong. S1 did not pay attention to the transaction priority here. S2 made mistakes when giving examples of situations where x is an even number in this expression. When he/she first thought of x as 10, he was able to say that he had to multiply 2 and 10 and showed that the result of the algebraic expression would be 25. However, he/she stated that 25 is an even number. He/she stated that if he/she had not multiplied 2 and 10, it would have shown 2x as 210, and that 2x+5 would have been 215. He/she stated that 215 cannot be an even number. Here, it is thought that S2 could not fully understand the concepts of even numbers and odd numbers in his/her mind. S2 thinks of two-digit numbers as even numbers. As a result of the numerical calculation, S3 showed that 2x+5 would be an odd number when he/she thought of x as an even number. He/she explained this situation by stating that the result of every number added by 5 will be an odd number. On the other hand, he/she could not explain the expression algebraically. S4 could only show numerically that when x is thought of as an even number 2, the result will be an odd number. S5 was able to show numerically that the expression represents odd numbers when he/she gave various even number values to x. He/she stated that in order to know that x is an even number, if x+2=4 is said, it can be understood that x will be an even number. However, he/she said that as a result of this process, x would always indicate the number 2, and stated that x could actually be a number other than 2.

DISCUSSION and CONCLUSIONS

KISS theory is based on the procept theory developed by Gray and Tall (1994). According to the KISS theory, which means conceptual operational symbolic process, symbolic expression gains importance in the transition from concrete thinking to abstract thinking in mathematical thinking. According to this theory, in mathematical thinking, which is handled in three stages, the individual first recognizes the process. It initially executes the action without realizing what it means. As a result of repetitive processes, the process begins to gain meaning for the individual. Then, he/she internalizes this process and the concept resulting from the process and shows it symbolically. In this study, the mathematical thinking processes of 6th grade students were discussed in terms of KISS theory in the context of the transition from arithmetic to algebra.

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The research was conducted with five sixth grade students who were receiving instruction on the subject of algebraic expressions. Studies (Carpenter & Levi, 2000; Cooper et al., 1997; Hersovics & Linchevski, 1994; Van Amerom, 2002) stated that the basis of arithmetic is the concept of number and that the origin of algebra is arithmetic. Therefore, in the research, first of all, the students' arithmetic knowledge was examined in terms of KISS theory. Their knowledge of the concepts of 5 and 4+3 was questioned. All students, except the student with the lowest mathematics achievement average, stated that the number 5 was formed as a result of the operational process. The student who could not explain only expressed the symbolic representation of 5 and said that it was an integer. On the other hand, all students were able to explain that the concept 4+3 both includes an operational process and expresses the concept 7 formed as a result of the process. Therefore, it can be said that students can think about all the processes in the KISS theory for a numerical concept. In concepts such as 5 and 4+3, it is expected that students think about the process and concept together. Students are in the sixth grade and according to Tall (2006), the concept of number is formed in the mind in the preschool period. However, the aim of this research is to reveal how students transition to abstract thinking in three stages according to the KISS theory. Therefore, it is seen that students think towards the process of symbolically expressing the concept as a result of operations and procedural processes in their mental activities.

During the transition from arithmetic to algebra, students knowledge of the concept of a+3b was questioned. Although the student with the lowest achievement level said that the letters represented numbers, he/she could not give correct explanations about the expression. Because he/she thought of letters as numbers, he/she focused on concrete thinking rather than abstract thinking. Similarly, the student with a high level of success could not explain a+3b even though he/she stated that the letters represented different numbers. Other students were able to make correct explanations by giving concrete examples or translating the expression into a verbal expression. In the process of transforming verbal expression into symbolic expression, students were generally able to write correct algebraic expressions. However, when explaining the expressions, they made concrete thinking by giving number values instead of unknowns. They had difficulty in transitioning to the abstract thinking process. Birgin and Demirören (2020) stated in their research that seventh and eighth grade students also had difficulties in learning algebraic expressions and had difficulty in the transition from arithmetic to algebra.

In the research, in the process where students were expected to think abstractly regarding algebraic expressions, they again turned to concrete thinking. While they were expected to explain algebraic expressions symbolically, they preferred to explain them by giving numerical values instead of unknowns. This is an expected situation since students in the 6th grade are new to symbolic expressions. Studies have reported (Biber & Argün, 2012; Yeşildere & Akkoç, 2011) that students tend to make trial and error in their mathematical thinking processes when they have difficulty. In fact, it is stated in the literature that when students are asked to provide proof, they find examples sufficient (Aylar, 2014; Çalışkan, 2012; Usta & Gökkurt Özdemir, 2018). Similarly, it has been stated that even high school students try to prove with numerical values and avoid algebraic expressions (Arslan & Yıldız, 2010). Therefore, lessons can include activities where students can improve their mental skills in transition to abstract thinking. Lessons can be planned by taking into account the mathematical thinking processes included in the KISS theory.

According to the KISS theory, which includes a three-stage thinking process, mathematical thinking involves the transition from concrete thinking to abstract thinking. In this research, the study was conducted with 6th grade students. In future research, KISS theory can be examined in more detail through studies covering the entire secondary school period. Additionally, in this research, KISS theory was discussed within the scope of algebra learning field. The functionality of the theory can also be examined in other areas of mathematics in the context of symbolic thinking processes.

REFERENCES

Açan, H. (2015). *Examining 8th graders' knowledge construction processes in transformation geometry* (Unpublished master's thesis). Dokuz Eylul University, Institute of Educational Sciences, Izmir.

Açıl, E. (2015). *Investigation of middle school 7rd graders abstraction processes of the concept of equation: APOS theory* (Unpublished doctoral dissertation). Ataturk University, Institute of Educational Sciences, Erzurum

Alkan, H. & Güzel, E. B. (2005). Development of mathematical thinking in the student teachers. *Gazi University Journal of Gazi Education Faculty*, 25 (3) , 221-236. Retrieved from https://dergipark.org.tr/tr/pub/gefad/issue/6755/90835

Alkan, H. & Tataroğlu Taşdan, B. (2011). Mathematical thinking through the eyes of prospective mathematics teachers at different grade levels. *Inonu University Journal Of The Faculty Of Education*, *12*(2), 107-137.

Altaylı Özgül, D. (2018). *Investigation of abstraction processes of secondary school students on the subject of polygons: RBC+C model* (Unpublished doctoral dissertation). Atatürk University, Institute of Educational Sciences, Erzurum.

Arslan, S. ve Yıldız, C. (2010). Reflections from the experiences of 11th graders during the stages of mathematical thinking. *Education and Science*, *35*(156), 17-31.

Aylar, E. (2014). *Examination of 7th grade students' ability on proving and their perception of proving* (Unpublished doctoral dissertation). Hacettepe University, Institute of Educational Sciences, Ankara.

Bağdat O., & Saban P. (2014). Investigation of the 8th grade students' algebraic thinking skills with solo taxonomy. *International Journal of Social Science*, 26 (2), 473-496.

Bahadır, K. (2020). *Reflections of mathematical thinking on the classroom environment: The case of 8th grade* (Unpublished master's thesis). Giresun University, Institute of Science, Giresun.

Baltacı, S. (2016). Examination of gifted students' probability problem solving process in terms of mathematical thinking. *Malaysian Online Journal of Educational Technology*, *4*(4), 18-35.

Baş, S. (2013). *An investigation of teachers' noticing of students' mathematical thinking in the context of a professional development program* (Unpublished doctoral dissertation). Middle East Technical University, Institute of Science, Ankara.

Bayraktar, F., Aydoğdu, M. & Tutak, T. (2022). Investigation of cognitive process steps of secondary school students: RBC+C model. *Journal of Education and Humanities: Theory and Practice*, *13*(25), 73-92.

Benli, F. B. & Gürtaş, K. (2021). Examination of secondary school 7th grade students' mathematical thinking and problem solving skills on rational numbers. *Pearson Journal Of Social Sciences & Humanities*, *6*(15), 552–562. https://doi.org/10.46872/pj.418

Biber, A., & Argün, Z. (2012). Investigating the mathematics pre-service teachers' construction of function of two variables limit concept. *Mersin University Journal of the Faculty of Education*, *8*(2), 56-66.

Birgin, O. & Demirören, K. (2020). Investigation of 7th and 8th grade students' performance about algebraic expressions. *PAU Journal of Education*, *50*, 99-117.

Bozkurt, A. & Topal, A. (2019). The examination of mathematical thinking of 6th grade students to solve process-constrained and process-open questions. *International Journal of Educational Studies in Mathematics*, *6*(2), 44-59.

Bulut, M. (2009). *The effect of computer algebra systems in the constructivist approach based cooperative learning environment on mathematical thinking and academic performance of students* (Unpublished doctoral dissertation). Gazi University, Institute of Educational Sciences, Ankara.

Burton, L. (1984). Mathematical thinking: The struggle for meaning. *Journal for Research in Mathematics Education*, *15*(1), 35-49.

Cai, J. (2000). Mathematical thinking involved in US and Chinese students' solving of process-constrained and process-open problems. *Mathematical Thinking and Learning*, *2*(4), 309-340.

Cai, J. (2003). Singaporean students' mathematical thinking in problem solving and problem posing: An exploratory study. *International Journal of Mathematical Education in Science and Technology*, *34*(5), 719–737.

Carpenter, T. P., & Levi, L. (2000). *Developing conceptions of algebraic reasoning in the primary grades*. Research Report, National Center for Improving Student Learning and Achievement in Mathematics and Science. Madison: University of Wisconsin. Retrieved from http://ncisla.wceruw.org/-publications/reports/RR-002.PDF.

Casey, S. & Amidon, J. (2020). Do you see what I see? Formative assessment of preservice teachers' noticing of students' mathematical thinking. *Mathematics Teacher Educator*, *8*(3), 88–104.

Chan, C.C, Tsui, M.S, Chan, M.Y.C. & Hong, J.H. (2002). Applying the structure of the observed learning outcomes (SOLO) taxonomy on student's learning outcomes: An empirical study. *Assessment and Evaluation in Higher Education*, *27*(6), 511-527.

Chin, E.T. & Tall, D. (2002) Proof as a formal procept in advanced mathematical thinking. u: Lin F-L. [ur.] *Proceedings of the International Conference on Mathematics: Understanding Proving and Proving to Understand* (pp. 212-221), Taipei, Taiwan: National Taiwan Normal University.

Cooper, T., Boulton-Lewis, G., Athew, B., Wilssi L. & Mutch, S. (1997). The transition arithmetic to algebra: Initial understandings of equals, operations and variable. *International Group for the Psychology of Matematics Education, 21*(2), 89-96.

Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches* (4nd ed.). Thousand Oaks, CA: Sage.

Çalışkan, Ç. (2012). *The interrelations between 8th grade class students' mathematics success and proving levels* (Unpublished master's thesis). Uludağ University, Institute of Educational Sciences, Bursa.

Çelik, D. (2007). *Analytical examination of the preservice teachers' algebraic thinking skills* (Unpublished doctoral dissertation). Karadeniz Technical University, Institute of Science, Trabzon.

Deniz, Ö. (2014). *Examination of 8th grade students' construction of the concept of slope based on realistic mathematics education in APOS framework* (Unpublished master's thesis). Anadolu University, Institute of Educational Sciences, Eskişehir.

Dinç, Y. & Yenilmez, K. (2022). An investigation of eighth grade students' knowledge construction process of square root numbers. *International Journal of Educational Studies in Mathematics*, *9*(2), 84-107.

Dreyfus, T. (1990). Advanced mathematical thinking. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition: A Research synthesis by the ınternational group for the psychology of mathematics education* (pp. 113-134). Cambridge, England: Cambridge University Press.

Eldekçi, S. (2019). *Revealing 7th grade middle school students' abstraction processes of variable concept with RBC model* (Unpublished master's thesis). Abant Izzet Baysal University, Institute of Educational Sciences, Bolu.

Ercire, Y.E. (2014). *Investigation of difficulties regarding the concept of irrational number* (Unpublished master's thesis). Dokuz Eylul University, Institute of Educational Sciences, İzmir.

Flake, M. W. (2014). *An investigation of how preservice teachers' ability to professionally notice children's mathematical thinking relates to their own mathematical knowledge for teaching* (Unpublished doctoral dissertation). The Graduate Faculty of the University of Kansas, Lawrence.

Fuson, K. C. & Hall, J. W. (1983). The acquisition of early number word meanings: A conceptual analysis and review. In H. P. Ginsburg (Ed.), *The Development of mathematical thinking* (pp. 49-107), New York: Academic Press.

Gelman, R. & Meck, E. (1986). The Notion of principle: The case of counting. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 29-58), Hillsdale: Lawrence Erlbaum Associates.

Goggins, L. L. (2007). *Eliciting elementary preservice teachers' mathematical knowledge for teaching using instructional tasks that include children's mathematical thinking* (Unpublished doctoral dissertation). The University of Delaware, Newark.

Gray, E. & Tall, D. (1994). Duality, ambiguity and flexibility: A proceptual view of simple arithmetic. *The Journal For Research in Mathematics Education*, *26*(2), 115-141.

Groth, R. E. & Bergner, J.A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning*, *8*(1), 37-63.

Günaydın, R. (2018). *Investigation in the APOS theoretical framework of the process of creating fraction concept in the 5th grade students'* (Unpublished master's thesis). Eskisehir Osmangazi University, Institute of Educational Sciences, Eskisehir.

Gürbüz, M. Ç. (2021). *Investigating secondary school students' abstraction processes of algebraic concepts* (Unpublished doctoral dissertation). Bursa Uludağ University, Institute of Educational Sciences, Bursa.

Gürbüz, M. K. (2018). *Investigation of the process of creating ratio-proportion concepts in the 7th grade students' activity based learning approach: APOS theory (*Unpublished master's thesis). Eskisehir Osmangazi University, Institute of Educational Sciences, Eskisehir.

Hannah, J., Stewart, S. & Thomas, M. O. J. (2016). Developing conceptual understanding and definitional clarity in linear algebra through the three worlds of mathematical thinking. *Teaching Mathematics and Its Applications: An International Journal of The IMA*, *35*(4), 216-235.

Hazar, D. (2021). *Construction of linear algebra concepts through 3d hologram-based learning* (Unpublished doctoral dissertation). Dokuz Eylul University, Institute of Educational Sciences, Izmir.

Henderson, P. B., Fritz, S. J., Hamer, J., Hitcher, L., Marion, B., Riedesel, C. & Scharf, C. (2002). Materials development in support of mathematical thinking. *The 7th Annual Conference on Innovation and Technology in Computer Science Education Working Group Report, ACM SIGCSE Bulletin*, *35*(2), 185–190.

Heng, M. A. & Sudarshan, A. (2013). "Bigger number means you plus!" – Teachers' learning to use clinical interviews to understand students' mathematical thinking. *Educational Studies in Mathematics*, *83*(3), 471-485.

Hersovics, N. & Linchevski, L. (1994). A cognative gap between arithmetic and algebra. *Educational Studies in Mathematics, 27*(1), 59-78.

Jacobs, V. R., Lamb, L. & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*(2), 169–202.

Kaplan, A. & Açıl, E. (2015). The investigation of the 4th grade secondary school students' construction processes in "inequality". *Journal of Bayburt Education Faculty*, *10*(1), 130-153.

Karakoca, A. (2011). *Using of mathematical thinking of sixth grade students in problem-solving* (Unpublished master's thesis). Eskisehir Osmangazi University, Institute of Educational Sciences, Eskisehir.

Karslıgil Ergin, G. (2015). *Students mathematical thinking in problem solving and problem posing* (Unpublished master's thesis). Gaziantep University, Institute of Educational Sciences, Gaziantep.

Keskin, M., Akbaba Dağ, S. & Altun, M. (2013). Comparison of 8 th and 11 th grade students behaviours at mathematical thinking. *Journal of Educational Science*, *1*(1), 33-50.

Kılıç, P., Tunç Pekkan, Z. & Karatoprak, R. (2013). The effects of using materials on mathematical thinking skills. *Journal of Theory and Practise in Education, 9*(4), 544-556.

Kidron, I. (2008). Abstraction and consolidation of the limit procept by means of instrumented schemes: The complementary role of three different frameworks. *Educational Studies in Mathematics*, *69*, 197-216.

Köse, O. (2018). *Determination of SOLO taxonomy levels of mathematics teacher candidates with high level saptial ability accordingto thinking structures* (Unpublished master's thesis). Selcuk University, Konya.

Kükey, E. (2018). *An investigation of mathematical thinking styles of middle school students with the opinions of teachers and pre-service teachers on this subject* (Unpublished master's thesis). Inönü University, Institute of Educational Sciences, Malatya.

Lane, C. P. (2005). *Mathematical thinking and the process of specializing* (Unpublished doctoral dissertation). M.S. University of Cincinnati, Cincinnati.

Lian, L. H., Yew, W. T. & Idrıs, N. (2010). Superitem test: An alternative assessment tool to assess students' algebraic solving ability. *International Journal for Mathematics Teaching and Learning, 185*, 1-15.

Liu, P. H. (2003). Do teachers need to incorporate the history of mathematics in their teaching?. *The Mathematics Teacher, 96*(6), 416-421.

Liu, P. H. & Niess, M. L. (2006). An exploratory study of college students' views of mathematical thinking in a historical approach calculus course. *Mathematical Thinking and Learning, 8*(4), 373-406.

Liu, Y. (2014). *Teachers' in-the-moment noticing of students' mathematical thinking: A case study of two teacher* (Unpublished doctoral dissertation). The University of North Carolina, Chapel Hill.

Lucas, U. & Mladenovic, R. (2008). The identification of variation in students' understandings of disciplinary concepts: The application of the SOLO Taxonomy within introductory accounting. *Higher Education, 58*(2), 257-283. doi:10.1007/s10734-009-9218-9

Ma'Moon, M.M. (2005). *Mathematical thinking and mathematics achievement of students in the year 11 scientific stream in Jordan* (Unpublished doctoral dissertation). The University of Newcastle, Australia.

Martínez Planell, R. & Trigueros, M. (2019). Using cycles of research in APOS: The case of functions of two variables. *The Journal of Mathematical Behavior, 53*, 1-22.

Mason, J., Burton, L. & Stacey, K. (2010). *Thinking mathematically* (Second Edition). London: Pearson Education Limited.

Miles, M, B. & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. (2nd ed). Thousand Oaks, CA: Sage.

MoNE (2018). *Mathematics Course Curriculum* (1st-8th Grades). Ankara: MEB.

Mudrikah, A. (2016). Problem-based learning associated by action-process-object-schema (APOS) theory to enhance students' high order mathematical thinking ability. *International Journal of Research in Education and Science (IJRES), 2*(1), 125-135.

Özdemir Baki, G. & Işık, A. (2018). Investigation of the noticing levels of teachers about students' mathematical thinking: a lesson study model. *Turkish Journal of Computer and Mathematics Education, 9*(1), 122-146.

Patton, M. Q. (2014). *Qualitative research and evaluation methods* (4. edition). Thousand Oaks, CA: Sage Publications.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton: Princeton University Press.

Rider, R.L. (2004). *The effect of multi-representational methods on students' knowledge of function concepts in developmental college mathematics* (Unpublished doctoral dissertation). Graduate Faculty of North Carolina State Universty, North Carolina.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sensemaking in mathematics. In Grouws, D. (Ed), *Handbook for research on mathematics teaching and learning* (pp. 334-370), Newyork: MacMillan.

Sezer, N. (2019). *Mathematical thinking processes and skills of secondary schoolstudents' longitudinal investigation* (Unpublished doctoral dissertation) Bursa Uludağ University, Institute of Educational Sciences, Bursa.

Sezgin Memnun, D. (2011). *The investigation of sixth grade students? construction of coordinate system and linear equation concepts of the analytical geometry using constructivism and realistic mathematics education* (Unpublished doctoral dissertation). Bursa Uludağ University, Institute of Educational Sciences, Bursa.

Stacey, K. (2006). What is mathematical thinking and why is it important? In *Progress report of the APEC project: Collaborative studies on innovations for teaching and learning mathematics in different cultures (II)-Lesson study focusing on mathematical thinking*, Tokyo: CRICED, University of Tsukuba.

Şefik, Ö. (2017). *Analysis of students' understanding of the concept of two variables functions by Apos theory* (Unpublished doctoral dissertation). Hacettepe University, Institute of Educational Sciences, Ankara.

Tabach, M. & Hershkowitz, R. (2002) Construction of knowledge and its consolidation: A case study from the early algebra classroom. In A. D. Cockburn & E. Nardi (Eds.) *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education, 4*(pp. 265-272). Norwich, United Kingdom: PME.

Tall, D. (2002). *Advanced mathematical thinking*. USA: Kluwer Academic Publishers.

Tall, D. (2006). A Theory of mathematical growth through embodiment, symbolism and proof. *Annales De Didactique Et De Sciences Cognitives, 11*, 195-215.

Tataroğlu Taşdan, B., Çelik, A. & Erduran, A. (2013). Examining the opinions of pre-service mathematics teachers on mathematical thinking and developing students' mathematical thinking. *Kastamonu Education Journal, 21*(4), 1487-1504.

Tohir, M., Maswar, M., Atikurrahman, M., Saiful, S. & Pradita, D. A. R. (2020). Prospective teachers' expectations of students' mathematical thinking processes in solving problems. *European Journal of Educational Research, 9*(4), 1735-1748. https://doi.org/10.12973/eu-jer.9.4.1735

Tsamir, P. & Dreyfus, T. (2002). Comparing infinite sets – A process of abstraction: The case of Ben. *Journal of Mathematical Behaviour, 21*, 1-23.

Türnüklü, A. (2000). A qualitative research technique that can be used effectively in educational research: Interview. *Educational Management: Theory & Practice, 6*(4), 543-559.

Türnüklü, E. & Özcan, B. (2014). The relationship between students' construction of geometric knowledge process based on rbc theory and van hiele geometric thinking levels: Case study. *Mustafa Kemal University Journal of Graduate School of Social Sciences, 11* (27), 295-316.

Tüzün, M. & Cihangir, A. (2020). Analysis of the relation between mathematical thinking stages and mathematics self efficacy of secondary school students. *Journal of Ahmet Kelesoğlu Education Faculty, 2*(2) , 210-228.

Ulaş, T. & Yenilmez, K. (2017). analyzing the eighth grade students' formation process of the identity concept. *International e-Journal of Educational Studies, 1*(2), 103-117.

Usta, N. ve Gökkurt Özdemir, B. (2018). Examination of algebraic thinking levels of middle school students. *Journal of Qualitative Research in Education, 6*(3), 427-453. DOI:10.14689/issn.2148-2624.1.6c3s20m

Uygun, T. (2020). Pre-Service middle level mathematics teachers' noticing of student mathematical thinking and teacher identity in the context of virtual experimentation. *International Journal of Mathematical Education in Science and Technology, 51*(7), 1098-1119.

Van Amerom, B. (2002). *Reinvention of early algebra: Developmental research on the transition from arithmetic to algebra* (Unpublished doctoral dissertation). University of Utrecht, The Netherlands.

Yeşildere, S. (2006). *The Investigation of mathematical thinking and knowledge construction processes of primary 6, 7 and 8th grade students who have different mathematical power* (Unpublished doctoral dissertation). Dokuz Eylul University, Institute of Educational Sciences, Izmir.

Yeşildere, S. & Akkoç, H. (2011). Pre-Service mathematics teachers' generalization processes of visual patterns. *Pamukkale University Journal of Education, 30*(30), 141-153.

Yeşildere, S. & Türnüklü, E. (2007). Examination of students' mathematical thinking and reasoning processes. *Ankara University Journal of Faculty of Educational Sciences (JFES), 40*(1), 181-213.

Yıldırım, A. & Şimşek, H. (2011). *Qualitative research methods in social sciences (8th edition).* Ankara: Seckin Publishing.

Yıldırım, D. & Yavuzsoy Köse, N. (2018). Mathematical thinking processes of secondary school students in polygon problems. *Bolu Abant Izzet Baysal University Journal of Faculty of Education, 18*(1), 605-633. doi:10.17240/aibuefd.2018..-362044.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd Edition). London: Sage Publications.

Wagner, S. H. & Walters, J. (1982). A longitudinal analysis of early number concepts. In G. E. Freeman (Ed.), *Action and thought: From sensorimotor schemes to symbolic operations* (pp. 137-161), Newyork: Academic Press.

Watson, A., Spyrou, P. & Tall, D. (2003). The relationship between physical embodiment and mathematical symbolism: The concept of vector. *The Mediterranean Journal of Mathematics Education, 1*(2), 73-97.

Zhao, H., Alexander, P. A. & Sun, Y. (2020). Relational reasoning's contributions to mathematical thinking and performance in Chinese elementary and middle-school students. *Journal of Educational Psychology, 113*(2), 279-303. doi:10.1037/edu0000595