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PRE-SERVICE TEACHERS' ANALYSIS SUCCESS AND SPATIAL VISUALIZATION SKILLS ABOUT ROTATIONAL OBJECTS[1]

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Abstract

The purpose of the research is to examine undergraduate program in mathematics education students' analysis success and spatial visualization skills about rotational objects. In the study, Explanatory Research Design was used. Area and Volume Calculation Scale about Rotational Objects (six items) and Spatial Visualization Scale about Rotational Objects (five items) were developed and applied by the researchers. In findings, while pre-service teachers were generally successful in calculating area and volume about rotational objects; they were more successful in area calculation than volume calculation. A significant difference was found between the participants' ability to calculate area and volume about rotational objects and their spatial visualization skills. In the content analysis, it was determined that the participants had difficulty in recognizing the names of axes, constructing rotational objects, and defining the rotational object in different positions.

Keywords: Rotational objects, analyses success, spatial visualization.

INTRODUCTION

Spatial ability is to understand the stance of an object in three-dimensional space and visualizing the new state that this object will take by rotating it in any direction (Gardner, 2011). Spatial visualization, which is the most complex component of spatial ability (Linn & Petersen, 1985), is to be able to make sense of geometric shapes, transform geometric shapes in the mind, and establish whole-part relationships (McGee, 1979). McGee (1979) calls the ability to bend, open-close, rotate and move an object in the mind as spatial visualization. In spatial visualization, there is a movement or change in the whole or a part of the object (McGee, 1979). According to Linn and Petersen (1985) spatial visualization is the ability to organize and use complex processes. Okagaki and Frensch (1996) put forward a similar idea, emphasizing that the ability to use information at more than one level in spatial visualization comes to the fore. While Strong and Smith (2002) define spatial visualization as being able to visualize the movements of three-dimensional objects in different directions, Olkun and Altun (2003) define both the movements of two- and three-dimensional objects and the new situations that they will take as a result of these movements. Kayhan (2005), on the other hand, draws attention to the whole-part relationship and defines the spatial visualization skill as being able to understand the new situations that occur as a result of the movement of the whole shape and its parts.

One of the issues related to spatial visualization is rotational objects. Rotational objects are the objects obtained by the rotation of the planar curve around a line in the same plane (Karakaş&Baydaş, 2008). It requires the ability to transform two-dimensional objects into three dimensions in the mind. For

[1] The article is produced from the first writer's doctoral thesis in consultation of the second author.

example, the knowledge of which three-dimensional object will be obtained by rotating a two-dimensional rectangle rotates around its long side by 360 degrees is a knowledge about rotational objects. Origami activities can be given as an example to rotational objects. Questions regarding these skills are included in exams for middle school, high school and university students. Knowledge of rotational objects is considered very important for teachers to prepare their students for both national and international exams and to continue their careers.

A rotational object, which is frequently used in fields such as mathematics, engineering, architecture and manufacturing, is formed by the rotation of a geometric shape or a curve around a certain point or line (Ayres&Mendelson, 2013). Karakaş and Baydaş (2008), on the other hand, define rotational objects as geometric shapes formed as a result of the plane curve rotating at a certain angle around a line in the same plane. Objects such as spheres, cylinders or cones are rotational objects. The surface surrounding the rotational object is called the rotational surface (Ayres&Mendelson, 2013). Calculating the area of rotational surfaces and calculating the areas and volumes of rotational objects is a subject related to the concept of integral, which is often considered as difficult to understand and is encountered by pre-service teachers in undergraduate program in mathematics education at many universities in many countries and in the exams taken after university education (Delice&Sevimli, 2011; Rasslan&Tall, 2002). In addition, the ability to calculate area and volume in rotational objects is a skill related to spatial visualization, which is one of the subcomponents of spatial ability.

The concept of integral and its applications together with the related rules is one of the subjects that university students generally have difficulty in learning (Misu, Budayasa, Lukito, Hasnawati&Rahim, 2019). There are two possible reasons for this situation: It can be said that the concept of integral is seen as an advanced operation and that the integral is a binary operation that includes both the derivative and is used in the calculation of area and volume (Maharaj, 2004). The use of different methods and formulas in area and volume calculations made via integral can also cause students to experience confusion and difficulty (Delice&Ergene, 2015; Kusumaningrum, Irfan, Agustito, Wijayanto&Ratih, 2019). When all these reasons are considered together, when teaching the concept of integral, algebraic representation, graphical representation and the process of forming a rotating object are considered together, more meaningful learning is expected and also it is expected that the connection between the concepts will be established more easily (Sevimli&Delice, 2010; Sealey, 2006; Maharaj, 2014).

There are some researches about these issues in the related literature. The study, conducted by Kusumaningrum, Irfan, Agustito, Wijayanto, and Ratih (2019), is a qualitative study examining a student's ability to draw function graphs in constructing rotational objects. In the 2018/2019 academic year, 44 students from the sixth semester studying at a private university in the Yogyakarta autonomous region of Indonesia were expected to answer the requested question within 20 minutes. In the question, they were asked to draw the rotational object formed by the rotation of the region bounded by the $y=1$, $x=0$ and $x=4$ lines of the $y= x2+2$ function around the x axis and calculate its volume. Since 43 students could not construct the rotational object correctly, they could not calculate its volume correctly. Since 73% of the students drew the $y=x-1$ function instead of the $y=1$ line, they could not construct the rotational object. Since only one student out of 44 could give the correct answer, the research was conducted with the student. In order to obtain more detailed information, in the interview held on the same question, the student was asked to draw the graph of this function, create the rotational object formed by its rotation around the axis, and calculate the volume of this rotational object using integral formulas. In this interview, which lasted 30-40 minutes, the reasons for the actions taken by the student were asked, the student was guided to think again by asking appropriate questions where he had difficulties or made mistakes, and it was ensured that he found the right answer. At the end of the research, it was observed that asking guiding questions to the students gradually improved their spatial ability. This research, which reveals very striking results about spatial ability and rotational objects, shows that the prerequisites for calculating the volumes of rotational objects is to draw the given function graph and construct the rotational object correctly.

The aim of the doctoral thesis study conducted by Mofolo-Mbokane (2011) is to determine the difficulties experienced by university students studying in South Africa in calculating the volumes of rotational objects. In this study, 40 students who were attending to Education Faculty, interviews were conducted with the students and observation notes were kept. At the end of the research, it was determined that the students had difficulties in thinking in three dimensions, making graphic drawings and making the operations.

Orton (1983) conducted interviews with 110 participants to examine students' understanding of the fundamental analysis course. Of the participants aged 16-22 (55 male-55 female), 60 of them were high school students between the ages of 16-18, while 50 of them were students between the ages of 18-22 studying at the university mathematics teaching department. A total of two sessions of one hour interviews were conducted with the students. During the interviews, a 38-questions scale including limit, derivative, integral and its applications was applied to the participants and their answers were evaluated with partial scoring. In the questions that the students had difficulty in answering, it was tried to imply the correct answer with the guiding questions. At the end of the research, it was determined that the most difficult questions for the students were to create rotational objects and to calculate the area and volume of the rotational objects. In order to develop a better understanding of the relevant subjects, it is recommended to start the subject with simple expressions, to explain the reasons for the rules and to continue the teaching processes by clearly demonstrating the relationship between algebraic representation, graphical representation and three-dimensional object's rotating processes.

Delice and Ergene (2015) examined the drawings and rotation skills of university students during the solution process of integral volume problems with an interpretive paradigm. A total of 101 students from the faculty of engineering (environmental, civil and mechanical engineering), the mathematics department of the faculty of science and literature, and the primary school mathematics teacher of the faculty of education participated in a state university in Istanbul. The seven-item open-ended integral volume problems test developed by the researchers was applied to the students and the answers were evaluated with the partial evaluation method. In the research findings, it was determined that about half of the university students did not answer the problems and the number of students who gave partial or complete correct answers to the problems was low. However, it was seen that more than half of the students had sufficient ability to draw and rotate shapes.

In this study, it is aimed to examine the skills of students studying in the undergraduate program in mathematics education to recognize and calculate area and volume about rotational objects. It is thought that the study will bring innovation to the literature due to the limited number of studies in the literature on recognizing the rotational objects that require spatial visualization skills of students from undergraduate program in mathematics education and examining the area and volume calculation in these objects. This study seeks answers to the following questions:

1-)What is the level of analysis success of the students' who were attending to undergraduate program in mathematics education about calculation area and volume of the rotational objects?

2-)What are the spatial visualization skills of students' who were attending to undergraduate program in mathematics education about rotational objects?

3-)Is there a relationship between the spatial visualization skills and analyses success about rotational objects of students' who were attending to undergraduate program in mathematics education?

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METHOD

In this research, Explanatory Research Design, in which qualitative and quantitative methods are used together, was used.

Participants

The sample of the study was the second year students who continued undergraduate program in primary mathematics education at a state university in Izmir at the 2022-2023 academic years. 78 students (55 girl and 23 boys) took Area and Volume Calculation Scale about Rotational Objects and 22 students who were randomly selected from the same sample (16 girls and 6 boys) took Spatial Visualization Scale about Rotational Objects.

Data Collection Tools and Process

Data were collected with two scales developed by the researchers. These scales are the Area and Volume Calculation Scale about Rotational Objects and the Spatial Visualization Scale about Rotational Objects. While the questions in the Area and Volume Calculation Scale about Rotational Objects (six items) are questions that require calculating area and volume about rotational objects; each question was evaluated between 0 and 4 points. The Spatial Visualization Scale about Rotational Objects (five items) (each question in the scale was scored between 0 and 3 points with the partial scoring method) aims to measure the skills of forming and recognizing rotational objects (cylinder, cone and sphere) in the middle school mathematics curriculum. While applying the Area and Volume Calculation Scale about Rotational Objects to the students and the data gathered analysed with quantitative methods; The Spatial Visualization Scale about Rotational Objects was applied by conducting semi-structured interviews with the students. Before starting the implementation phase, a pilot study of the scales was conducted with 17 students (10 girls-7 boys) who were attending to mathematics teaching program at a state university in İzmir at the second semester of the 2021-2022 academic years. Scales were found reliable and valid according to the pilot study.

RESULTS

The first research question is “What is the level of analysis success of the students’ who were attending to undergraduate program in mathematics education about the area and volume of the rotational objects?” In this direction, primarily descriptive statistics were applied to the data collected from the Area and Volume Calculation Scale about Rotational Objects. In the descriptive statistics tests, the lowest and highest scores taken from the scale, the arithmetic mean and standard deviations of the scores were examined. Table 1 shows the descriptive statistical analysis results of the Area and Volume Calculation Scale about Rotational Objects.

Table 1. Descriptive statistical analysis results of area and volume calculation scale about rotational objects

Questions	Participants	Minimum Score	Maximum Score	Standard Deviation	Arithmetic Mean
1.question	78	0	4	1,06	3,56
2.question	78	0	4	1,38	2,42
3.question	78	0	4	1,07	3,33
4.question	78	0	4	0,92	3,47
5.question	78	0	4	1,06	3,44
6.question	78	2	4	0,78	3,55
Total Score	78	4	24	4,09	19,79

When the arithmetic means of each question were examined, it was seen that the students generally got high scores. It has been determined that at the second question the pre-service teachers were successful at least. Area and Volume Calculation Scale about Rotational Objects has a two-factor structure as

Area Calculation and Volume Calculation. The first, second, third and fourth questions are questions about volume calculation in rotational objects, and the arithmetic mean of the scores obtained by the participants from these questions was determined as 3,198. The fifth and sixth questions are about measuring area calculation skills in rotational objects, and the arithmetic mean of the scores obtained by the pre-service teachers from these questions was determined as 3,495. Therefore, it can be said that the pre-service teachers participating in the research were more successful in calculating area in rotational objects than in calculating volume in rotational objects. In addition, the arithmetic mean of all the scores was 19,79. It could be said that pre-service teachers were generally successful at the Area and Volume Calculation Scale about Rotational Objects.

In order to determine the item difficulty values of the items in the Area and Volume Calculation Scale about Rotational Objects, item difficulty coefficients were calculated. Table 2 shows the item difficulty coefficients of the items in the Area and Volume Calculation Scale about Rotational Objects. When Table 2 is examined, it is seen that the pre-service teachers can generally solve the questions easily. When Table 1 and Table 2 are evaluated together, it is determined that the second question was the question that pre-service teachers had the most difficulty and showed the least success.

Table 2. Item difficulty coefficients of the area and volume calculation scale about rotational objects

Questions	Item Difficulty Coefficients
1.question	.89
2.question	.605
3.question	.83
4.question	.86
5.question	.86
6.question	.88

The second research problem is "What are the spatial visualization skills of students' who were attending to undergraduate program in mathematics education about rotational objects?" In line with this research question, the Spatial Visualization Scale about Rotational Objects was applied to the participants by interview method. The spatial visualization skills and thinking processes of pre-service teachers were examined in depth by using the explanatory research method. It is aimed to evaluate the skills of constructing and defining rotational objects and visualizing their positions in the mind. Since it was desired to compare the spatial visualization skills of the pre-service teachers with the scores they got from the Area and Volume Calculation Scale about Rotational Objects, the Spatial Visualization Scale about Rotational Objects was also scored according to the correct answers given, and content analysis was also performed on these data.
One of the analyses made using quantitative techniques on the Spatial Visualization Scale about Rotational Objects is descriptive statistics. Table 3 shows the results of the descriptive statistical analysis of the Spatial Visualization Scale about Rotational Objects.

Table 3. Results of descriptive statistics analyses of spatial visualization scale about rotational objects

Questions	Minimum Score	Maximum Score	Arithmetic Mean	Standard Deviation
1.question	1	55	4,78	10,97
2.question	1	55	4,78	10,967
3.question	1	50	4,35	9,98
4.question	0	10	0,87	2,22
5.question	0	31	2,70	6,29

In order to examine the difficulty levels of the teacher candidates in the questions, the item difficulty coefficients were also examined. Table 4 shows the item difficulty coefficients of the Spatial Visualization Scale about Rotational Objects.

Table 4. Item difficulty coefficients of spatial visualization scale about rotational objects

Items	1.question	2.question	3.question	4.question	5.question
Item Difficulty Coefficients	.83	.83	.75	.15	.46

When Table 3 and Table 4 are evaluated together, it can be seen that the question in which the pre-service teachers had the most difficulty and therefore the least success was seen in the fourth question. While the pre-service teachers showed approximately the same success in the first, second and third questions, they showed relatively less success in the fifth question. The difficulties of pre-service teachers, their wrong answers and thinking processes were examined in depth with a detailed content analysis on the Spatial Visualization Scale about Rotational Objects. In the content analysis, the answers given by the pre-service teachers to the questions, participants' notes and researcher notes were examined in detail and it was tried to reveal the thinking processes of the pre-service teachers. In this part of the study, each question of the Spatial Visualization Scale about Rotational Objects, which consists of five questions, was handled one by one, different thinking processes of pre-service teachers were examined in detail for each question, and statistical analyses related to their thinking processes were given in tables for each question.

When the answers given by the teacher candidates to all the questions were evaluated in general, it was determined that the thinking processes were gathered around certain headings. These headings were determined as not being able to recognize the axis names (symmetry axis, x=0 and y=0 axis), not being able to construct the rotational object, not being able to define the rotational object in different positions, not being able to determine the edge lengths of the rotational object and showing all processes correctly. In line with these determined headings, each question was examined in detail.

In the first question of the Spatial Visualization Scale about Rotational Objects, an isosceles triangle was described. This depicted isosceles triangle was asked to be drawn, and the object formed when the triangle was rotated around of its symmetry axis. Besides, it was asked to draw and describe the rotational object together with its edge lengths. Table 5 shows the thinking processes of the participants in the first question of the Spatial Visualization Scale about Rotational Objects, the total number of participants, the number and percentage of participants showing the relevant thinking processes. Percentages of participants showing the relevant thinking process are given by rounding it to the nearest integer.

Table 5. Statistics of content analysis of the first question of spatial visualization scale about rotational objects

Thinking Processes	Participants	The Number of Participants Showing Relevant Thinking Processes	The percentage of Participants Showing Relevant Thinking Processes
Not Being Able to Recognize the Axis Names	22	3	% 14
Not Being Able to Construct the Rotational Object	22	3	% 14
Not Being Able to Define the Rotational Object in Different Positions	22	0	% 0
Not Being Able to Determine the Edge Lengths of the Rotational Objects	22	7	% 32
Showing All Processes Correctly	22	14	% 64

In the first question, it was requested to determine the symmetry axis of the isosceles triangle described, but it was seen that some participants confused the axis of symmetry with the coordinate axes. Therefore, these participants could not create the desired rotational object. In the related question, the rotational object consisting from the isosceles triangle was a prototype cone shape. Therefore, in this question, there was no problem related to the thinking process of not being able to recognize rotational objects in different positions. The mistake that the participants usually made in the question is not being able to determine the edge lengths of the rotational object correctly which is determining the radius length of the rotational object cone. It is thought that it may be caused by not thinking that the radius of the cone should be half of the base length of the triangle. It has been seen that it is to show the faulty thinking process in which the entire side length of the triangle will turn into a peripheral length. It was determined that the majority of the participants were able to show all their thinking processes correctly.

In the second question of the Spatial Visualization Scale about Rotational Objects, a rectangle was described and it was asked to define the rotational object formed by rotating this rectangle around one of its symmetry axes. Table 6 shows the thinking processes and related statistics in the second question.

Table 6. Statistics of content analysis of the second question of spatial visualization scale about rotational objects

Thinking Processes	Participants	The Number of Participants Showing Relevant Thinking Processes	The percentage of Participants Showing Relevant Thinking Processes
Not Being Able to Recognize the Axis Names	22	3	% 14
Not Being Able to Construct the Rotational Object	22	1	% 5
Not Being Able to Define the Rotational Object in Different Positions	22	0	% 0
Not Being Able to Determine the Edge Lengths of the Rotational Objects	22	7	% 32
Showing All Processes Correctly	22	12	% 55

It was determined that some of the students who had difficulty in recognizing the names of the axis in the first question of the Spatial Visualization about Rotational Objects scale had difficulties in recognizing the names of the axis and constructing the desired rotational object in the second question. Two pre-service teachers who misunderstood the axis names correctly guessed that by rotating the rectangle a cylinder would be formed. For this reason, the answers of these participants were evaluated only in the process of not being able to recognize the names of the axis. In the second question, although about half of the pre-service teachers were able to perform all thinking processes correctly; the total success in the second question was lower than the first question. In the second question, pre-service teachers had difficulty mostly at determining the edge lengths of the rotational object which was a cylinder. In the second question, it was determined that the pre-service teachers could not accurately visualize the process in which the axis of symmetry of the rectangle divides the side of the rectangle into two, and this formed part turns into the radius of the cylinder.

In the third question of the Spatial Visualization Scale about Rotational Objects, a trapezoid was described, and the participants were asked to draw this trapezoid, rotate it around the axis of symmetry, and describe the resulting rotational object. The correct answer should be frustum of a cone. Table 7 shows the statistics of content analysis of the third question.

Table 7. Statistics of content analysis of the third question of spatial visualization scale about rotational objects

Thinking Processes	Participants	The Number of Participants Showing Relevant Thinking Processes	The percentage of Participants Showing Relevant Thinking Processes
Not Being Able to Recognize the Axis Names	22	4	% 18
Not Being Able to Construct the Rotational Object	22	4	% 18
Not Being Able to Define the Rotational Object in Different Positions	22	6	% 27
Not Being Able to Determine the Edge Lengths of the Rotational Objects	22	5	% 23
Showing All Processes Correctly	22	11	% 50

In the third question of the Spatial Visualization Scale about Rotational Objects, it was determined that half of the pre-service teachers were able to show all thinking processes correctly. Compared to the first and second questions, in the third question it was seen that the number of pre-service teachers who showed all thinking processes correctly was less. Four of the pre-service teachers had difficulties both in drawing the axis of symmetry and in constructing the rotational object. Six pre-service teachers, who showed the thinking process of not being able to recognize the rotating object in different positions, either could not define the rotational object because s/he could not form it, or stated that they did not know the name of the rotational object despite being able to construct it correctly.

In the fourth question of the Spatial Visualization about Rotational Objects Scale, they were asked to draw and describe the rotational object that will be formed by rotating the region bounded by the y=2x line with the x=0 and x=3 lines around the y axis. Here, the pre-service teachers were expected to first determine the region to be rotated.

In the fourth question of the Spatial Visualization Scale about Rotational Objects, it was seen that the majority of the pre-service teachers could not know the axes' correct names. Therefore they could not correctly identify the region to be rotated and construct the rotational object correctly. It was determined that the most obvious reason for this situation was that the pre-service teachers thought the x=0 axis as the x axis (the correct answer is the y axis) and continued the question with an erroneous solution in that way. This problematic situation showed once again how important the use of mathematical language is.

In the fourth question of the Spatial Visualization Scale about Rotational Objects, it was determined that the number of participants who showed all the thinking processes correctly was quite low. The fourth question had the least correct answers among all questions. In addition to the erroneous thought process of not being able to recognize the axis names, it was seen that the erroneous thinking process of not being able to recognize the rotational object in different positions was also the cause of this situation. Because the desired area was a right triangle, and when this triangle was rotated around the y-axis, a cone whose position was exactly opposite to the prototype cone should be obtained. The ability to recognize rotational objects in different positions, which is the most prominent feature of the concept of spatial visualization, shows itself quite well here. Pre-service teachers are expected to know that its name does not change even if the cone turns upside down. It was observed that some pre-service teachers named the changing cone as an inverted cone instead of calling it a cone. This situation once again emphasizes the importance of both spatial visualization skills and the use of mathematical language. Table 8 shows the statistics of content analysis of the fourth question.

Table 8. Statistics of content analysis of the fourth question of spatial visualization scale about rotational objects

Thinking Processes	Participants	The Number of Participants Showing Relevant Thinking Processes	The percentage of Participants Showing Relevant Thinking Processes
Not Being Able to Recognize the Axis Names	22	18	% 82
Not Being Able to Construct the Rotational Object	22	18	% 82
Not Being Able to Define the Rotational Object in Different Positions	22	3	% 17
Not Being Able to Determine the Edge Lengths of the Rotational Objects	22	1	% 5
Showing All Processes Correctly	22	1	% 5

In the fourth question of the scale, most of the pre-service teachers could not form the correct rotational object because they confused the names of the axis. On the other hand, one of the other pre-service teachers was able to answer the question correctly, one of them stated the cone height incorrectly even though he was able to form the rotational object correctly, and 3 pre-service teachers made a wrong naming about the rotational object even though they determined the edge lengths correctly when the rotational object was in different positions.

In the fifth and last question of the Spatial Visualization Scale about Rotational Objects, students were asked to draw and describe the rotational object obtained by rotating the region bounded by the x=0 and x=3 lines of the f(x)= $\sqrt{(9-x^2)}$ function around the x axis. The region to be rotated was a quarter circle formed between the points x=0, x=3 and y=3. The rotational object to be formed was a hemisphere located in the first and fourth regions. Table 9 shows the statistics of content analysis of the fifth question of Spatial Visualization Scale about Rotational Objects

Table 9. Statistics of content analysis of the fifth question of spatial visualization scale about rotational objects

Thinking Processes	Participants	The Number of Participants Showing Relevant Thinking Processes	The percentage of Participants Showing Relevant Thinking Processes
Not Being Able to Recognize the Axis Names	22	0	% 0
Not Being Able to Construct the Rotational Object	22	12	% 56
Not Being Able to Define the Rotational Object in Different Positions	22	2	% 9
Not Being Able to Determine the Edge Lengths of the Rotational Objects	22	1	% 5
Showing All Processes Correctly	22	7	% 32

In the fifth question of the Spatial Visualization Scale about Rotational Objects, it is seen that there is no problem with not being able to recognize the axis names. It is estimated that this situation arises from the fact that it is sufficient to know only the x and y axes in the question. However, it was determined that more than half of the participants could not construct the desired rotational object. While two of the participants drew the position of the desired rotational object incorrectly; one of them could not determine the radius length of the formed hemisphere correctly. The number of participants who showed all thinking processes correctly was seven.

In the content analysis conducted on the Spatial Visualization Scale about Rotational Objects, when the subjects that pre-service teachers had difficulty in were evaluated in general, it was determined that the pre-service teachers generally knew the prototype (typical) positions of the rotational objects, but they had difficulty in defining it when this rotational object changed its position or direction. Another problem identified in the answers of the pre-service teachers is the ignorance of the concepts of "frustum of cone" and "hemisphere". Some of the pre-service teachers stated that although they could create the desired frustum of cone rotational object from the trapezoidal geometric shape described in the question, they did not know the name of this object. In this question, pre-service teachers answered, “It looks like a cone, but I know it isn't.”, “This object looks like a lamp, but I don't know its correct name.” or “I don't know the name of this geometric object.” Similarly, in the question of obtaining a hemisphere rotational object from a quarter circle, even if the pre-service teachers predicted that a hemisphere would be obtained from a quarter circle, they could not correctly draw the position of the hemisphere in the coordinate system, and there was a misdirection towards prototype drawings; the position of the hemispherical rotational object obtained by rotating this region around the x-axis could not be determined correctly. After realizing that a hemispherical rotational object would be obtained, the position of the object was ignored, and the idea of prototype object dominated.

Another common mistake made by pre-service teachers is to perceive the x=0 line as the x-axis and not being able to correctly determine the area to be rotated. It was seen that the most common mistake made by the teacher candidates in the interviews was due to this situation. Many of the participants could not correctly determine the area to be rotated by making the wrong solution, could not create the desired rotational object, and therefore could not define the rotational object correctly.

When the subjects that pre-service teachers were strong on the Spatial Visualization Scale about Rotational Objects, it is seen that more than half of the participants performed above the average. It has been determined that the pre-service teachers are generally successful in the creation of cone and cylinder rotational objects, which are among the basic acquisitions of the middle school mathematics curriculum, and in determining the edge lengths. In addition, almost all pre-service teachers were able to draw the described geometric objects correctly. The number of pre-service teachers who can complete all processes correctly is quite low. In summary, the strongest subject of the pre-service teachers in the Spatial Visualization Scale about Rotational Objects was being able to accurately draw the described geometric shape and it has been determined that the weakest issue was not being able to recognize the axis names (thinking the x = 0 axis as the x axis) and not being able to visualize the rotational objects in different positions in their minds.

The third research problem is “Is there a relationship between the spatial visualization skills and analyses success about rotational objects of students’ who were attending to undergraduate program in mathematics education?” When the answers given by the pre-service teachers who participated in the interviews are examined, it is seen that the pre-service teachers who received high scores in the Area and Volume Calculation Scale about Rotational Objects were able to give correct answers and perform the correct thinking processes in the Spatial Visualization Scale about Rotational Objects. Similarly, it was determined that pre-service teachers who showed low success in the Area and Volume Calculation Scale about Rotational Objects also showed low success in the Spatial Visualization Scale about Rotational Objects.

When the relationship between Area and Volume Calculation Scale about Rotational Objects and Spatial Visualization Scale about Rotational Objects scores was desired to be analysed statistically, the Wilcoxon Signed Rank Test, one of the non-parametric tests, was applied. Table 10 shows the test results.

Table 10. Wilcoxon signed rank test results

Values	Total	Mean of the Ranks	Total Runks	Z	p
Negative Ranks	20[a]	12,5	250	-4,013[b]	.000*
Pozitive Ranks	2[b]	1,5	3		
Equal Ranks	0[c]				
Total	22				

The statistically significant difference was found as a result of this test. It also supports the results of the content analysis. In other words, the participants' Area and Volume Calculation Scale scores about Rotational Objects and Spatial Visualization Scale about Rotational Objects scores showed parallelism.

DISCUSSION and CONCLUSIONS

When we look at the area and volume calculation analysis successes of undergraduate program in mathematics education students, it was seen that the pre-service teachers generally showed success above the average. It was determined that most of the pre-service teachers did not have any difficulties in writing the formulas for calculating the area and volume of the rotational objects and solving the questions in line with these formulas.

When the spatial visualization skills of undergraduate program in mathematics education students about rotational objects were examined, it was seen that approximately half of the pre-service teachers were able to successfully complete the problem-solving processes. The studies of Linn and Peterson (1985), who stated that spatial visualization is the most complex component among the spatial ability sub-components, also support this result. Alike the study conducted by Delice and Ergene (2015), approximately half of the students in the elementary mathematics teaching undergraduate program were successful in spatial visualization.

In the content analysis on the spatial visualization scale data, it was determined that the pre-service teachers showed faulty in thinking processes such as not being able to recognize the axis names, not being able to construct the rotational object, not being able to define the rotational object in different positions, and not being able to determine the edge lengths of the rotational object. In addition to these, the number of pre-service teachers who showed all their thinking processes correctly was also determined. In the study conducted by Kusumaningrum, Irfan, Agustito, Wijayanto, and Ratih (2019), the spatial visualization skills of the students in the mathematics teaching department were examined and it was determined that the pre-service teachers showed thinking processes such as not being able to recognize the names of the axis, not constructing the rotational object, and not being able to determining the desired region to be rotated. In the study conducted by Mofolo-Mbokane (2011), it was determined that pre-service teachers had difficulties in constructing the rotational objects and determining the region to be rotated. In the study conducted by Orton (1983), it was seen that the thinking process that teacher candidates had the most difficulty was constructing the rotational object. Quite similar to these studies, in this study, it was determined that some pre-service teachers had difficulties in recognizing the names of the axis, and therefore they could not correctly determined the desired area to be rotated. However, the spatial visualization thinking processes revealed in this research are more comprehensive and more explanatory than the studies that the spatial visualization thinking processes put forward in the current literature. It is thought that this situation contribute significantly to the related literature.

When the relationship between pre-service teachers' skills in calculating area and volume about rotational objects and spatial visualization skills was examined, it was seen that these skills were parallel to each other. In other words, pre-service teachers who were successful in calculating area and volume in rotational objects were also successful in spatial visualization in rotational objects. This has been demonstrated by both content analysis and statistical tests.

Pre-service teachers are generally expected to be able to write and apply the area and volume formulas correctly in calculating area and volume about rotational objects. In this study, it was seen that more than half of the teacher candidates were successful in general. However, the main point to be considered here has been revealed by the content analysis. In the content analysis, it was determined that some pre-service teachers who were successful in calculating area and volume made mistakes in axis names in other questions, could not recognize the rotational object when its position changed, or made mistakes in determining the edge lengths. While area and volume calculation are formula-based and more related to the ability to operate; spatial visualization is a more complex and relatively difficult skill that involves abstract thinking and requires the ability to move a rotational object flexibly in the mind. For this reason, a pre-service teacher can easily calculate the area and volume of a rotational object, but when asked to construct this rotational object and to determine its position and edge lengths, s/he may have difficulties. When these reasons are considered together, it is thought that the content analysis is very useful and produces striking results. In addition, it is thought that the idea that different thinking processes may emerge when other samples are studied.

Another important issue revealed by the content analysis was the importance of using mathematical language. Although some pre-service teachers were able to construct the rotational object correctly, it was observed that they gave different names when its position was not in its generally known position; that is, when it was different from the typical or prototype rotational object. Or, when they were asked to explain how they thought in the process of constructing the rotational object, it was determined that they made some mathematical language mistakes. Since this situation will cause deficiencies and mistakes in their learning and it is predicted that this situation will lead to incomplete and erroneous teaching when they will teach. It is thought that it is very important and necessary to give importance to the correct use of mathematical language in rotational objects as in all other subjects.

Rotational objects and the concept of integral, which are quite abstract concepts, are difficult to understand by students, difficult to visualize in minds and they include complex operations (Delice & Ergene, 2015). For this reason, teaching the subject of rotational objects has gained importance. Similar results were obtained in a study conducted by Maharaj (2014), and it was determined that the reason for this situation might be due to the inability of pre-service teachers to construct appropriate diagrams for the concept of integral. Alejandro and Liliana (2009) stated that it would be more beneficial to teach the concept of integral and rotational objects with technology supported teaching method. 3d visualization programs also facilitate students' visualization processes. It is foreseen that it will be easier to concretize abstract concepts with the help of technology. In addition, it is estimated that it will be beneficial for teacher candidates to realize conceptual learning about rotational objects and to make the explanations with a relational method in a cause-effect relationship.

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